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CENTRE FOR DISTANCE AND ONLINE EDUCATION

(CDOE)

BACHELOR OF COMMERCE

SEMESTER - I

CORE – I: OPERATION RESEARCH

(Candidates admitted from 2024 onwards)

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UNIT I

Unit Objectives

Operations Research (OR) is a discipline that applies advanced analytical methods to help make better decisions. By employing techniques from mathematics, statistics, and computer science, OR aims to optimize the performance of complex systems in various fields such as business, engineering, healthcare, and transportation. The primary goal of OR is to provide a rational basis for decision-making by seeking the best possible solutions to problems involving the allocation of limited resources. One of the most fundamental and widely used methodologies within OR is Linear Programming (LP). Linear Programming is a mathematical technique used for optimizing a linear objective function, subject to a set of linear equality and inequality constraints. LP problems are characterized by their ability to model real-world situations where resources must be allocated efficiently under certain restrictions. Typical applications include maximizing profit, minimizing cost, and optimizing production schedules. The significance of LP lies in its simplicity and versatility, allowing it to address a broad spectrum of optimization problems with precision and clarity. By understanding the principles of Operations Research and mastering the formulation and solution of Linear Programming problems, decision-makers can significantly enhance their ability to achieve optimal outcomes in various operational contexts.

1. INTRODUCTION

The British/Europeans refer to "operational research", the Americans to "operations research" - but both are often shortened to just "OR" - which is the term we will use.

Another term which is used for this field is "management science" ("MS"). The Americans sometimes combine the terms OR and MS together and say "OR/MS" or "ORMS". Yet other terms sometimes used are "industrial engineering" ("IE") and "decision science" ("DS"). In recent years there has been a move towards a standardization upon a single term for the field, namely the term "OR".

Operation Research is a relatively new discipline. The contents and the boundaries of the OR are not yet fixed. Therefore, to give a formal definition of the term Operations Research is a difficult task. The OR starts when mathematical and quantitative techniques are used to substantiate the decision being taken. The main activity of a manager is the decision making. In our daily life we make the decisions even without noticing them. The decisions are taken simply by common sense, judgment and expertise without using any mathematical or any other model in simple situations. But the decision we are concerned here with are complex and heavily responsible. Examples are public transportation network planning in a city having its own layout of factories, residential blocks or finding the appropriate product mix when there exists a large number of products with different profit contributions and production requirement etc.

Operations Research tools are not from any one discipline. Operations Research takes tools from different discipline such as mathematics, statistics, economics, psychology, engineering etc. and combines these tools to make a new set of knowledge for decision making. Today, O.R. became a professional discipline which deals with the application of scientific methods for making decision, and especially to the allocation of scarce resources. The main purpose of O.R. is to provide a rational basis for decisions making in the absence of complete information, because the systems composed of human, machine, and procedures may do not have complete information.

Operations Research can also be treated as science in the sense it describing, understanding and predicting the systems behaviour, especially man-machine system. Thus O.R. specialists are involved in three classical aspect of science, they are as follows:

i). Determining the systems behaviour

ii). Analyzing the systems behaviour by developing appropriate models

iii). Predict the future behaviour using these models

The emphasis on analysis of operations as a whole distinguishes the O.R. from other research and engineering. O.R. is an interdisciplinary discipline which provided solutions to problems of military operations during World War II, and also successful in other operations. Today business applications are primarily concerned with O.R. analysis for the possible alternative actions. The business and industry befitted from O.R. in the areas of inventory, reorder policies, optimum location and size of warehouses, advertising policies, etc.

As stated earlier defining O.R. is a difficult task. The definitions stressed by various experts and Societies on the subject together enable us to know what O.R. is, and what it does.

They are as follows:

1.According to the Operational Research Society of Great Britain Operational Research is the attack of modern science on complex problems arising in the direction and management of large systems of men, machines, materials and money in industry, business, government and defense. Its distinctive approach is to develop a scientific model of the system, incorporating measurements of factors such as change and risk, with which to predict and compare the outcomes of alternative decisions, strategies or controls. The purpose is to help management determine its policy and actions scientifically.

2.Randy Robinson stresses that Operations Research is the application of scientific methods to improve the effectiveness of operations, decisions and management. By means such as analyzing data, creating mathematical models and proposing innovative approaches, Operations Research professionals develop scientifically based information that gives insight and guides decision- making. They also develop related software, systems, services and products.

3.Morse and Kimball have stressed O.R. is a quantitative approach and described it as " a scientific method of providing executive departments with a quantitative basis for decisions regarding the operations under their control".

4. Saaty considers O.R. as tool of improving quality of answers. He says, "O.R. is the art of giving bad answers to problems which otherwise have worse answers".

5.Miller and Starr state, "O.R. is applied decision theory, which uses any scientific, mathematical or logical means to attempt to cope with the problems that confront the executive, when he tries to achieve a thorough-going rationality in dealing with his decision problem".

6.Pocock stresses that O.R. is an applied Science. He states "O.R. is scientific methodology (analytical, mathematical, and quantitative) which by assessing the overall implication of various alternative courses of action in a management system provides an improved basis for management decisions"

1.2 History of Operations Research

Operation Research is a relatively new discipline. Whereas 70 years ago it would have been possible to study mathematics, physics or engineering (for example) at university it would not have been possible to study Operation Research, indeed the term O.R. did not exist then. It was really only in the late 1930's that operational research began in a systematic fashion, and it started in the UK. As such it would be interesting to give a short history of O.R.

1936

Early in 1936 the British Air Ministry established Bawdsey Research Station, on the east coast, near Felixstowe, Suffolk, as the centre where all pre-war radar experiments for both the Air Force and the Army would be carried out. Experimental radar equipment was brought up to a high state of reliability and ranges of over 100 miles on aircraft were obtained.

It was also in 1936 that Royal Air Force (RAF) Fighter Command, charged specifically with the air defense of Britain, was first created. It lacked however any effective fighter aircraft - no Hurricanes or Spitfires had come into service - and no radar data was yet fed into its very elementary warning and control system.

It had become clear that radar would create a whole new series of problems in fighter direction and control so in late 1936 some experiments started at Biggin Hill in Kent into the effective use of such data. This early work, attempting to integrate radar data with ground based observer data for fighter interception, was the start of OR.

1937

The first of three major pre-war air-defence exercises was carried out in the summer of 1937. The experimental radar station at Bawdsey Research Station was brought into operation and the information derived from it was fed into the general air-defense warning and control system. From the early warning point of view this exercise was encouraging, but the tracking information obtained from radar, after filtering and transmission through the control and display network, was not very satisfactory.

1938

In July 1938 a second major air-defense exercise was carried out. Four additional radar stations had been installed along the coast and it was hoped that Britain now had an aircraft location and control system greatly improved both in coverage and effectiveness. Not so! The exercise revealed, rather, that a new and serious problem had arisen. This was the need to coordinate and correlate the additional, and often conflicting, information received from the additional radar stations. With the outbreak of war apparently imminent, it was obvious that something new - drastic if necessary - had to be attempted. Some new approach was needed.

(military) OPERATIONS] was coined as a suitable description of this new branch of applied science. The first team was selected from amongst the scientists of the radar research group the same day.

1939

In the summer of 1939 Britain held what was to be its last pre-war air defence exercise. It involved some 33,000 men, 1,300 aircraft, 110 antiaircraft guns, 700 searchlights, and 100 barrage balloons. This exercise showed a great improvement in the operation of the air defence warning and control system. The contribution made by the OR team was so apparent that the Air Officer Commander-in-Chief RAF Fighter Command (Air Chief Marshal Sir Hugh Dowding) requested that, on the outbreak of war, they should be attached to his headquarters at Stanmore in north London.

Initially, they were designated the "Stanmore Research Section". In 1941 they were redesignated the "Operational Research Section" when the term was formalised and officially accepted, and similar sections set up at other RAF commands.

1940

On May 15th 1940, with German forces advancing rapidly in France, Stanmore Research Section was asked to analyses a French request for ten additional fighter squadrons (12 aircraft a squadron - so 120 aircraft in all) when losses were running at some three squadrons every two days (i.e. 36 aircraft every 2 days). They prepared graphs for Winston Churchill (the British Prime Minister of the time), based upon a study of current daily losses and replacement rates, indicating how rapidly such a move would deplete fighter strength. No aircraft were sent and most of those currently in France were recalled.

This is held by some to be the most strategic contribution to the course of the war made by OR (as the aircraft and pilots saved were consequently available for the successful air defense of Britain, the Battle of Britain).

1941 onward

In 1941, an Operational Research Section (ORS) was established in Coastal Command which was to carry out some of the most well-known OR work in World War II.

The responsibility of Coastal Command was, to a large extent, the flying of long-range sorties by single aircraft with the object of sighting and attacking surfaced U-boats (German submarines). The technology of the time meant that (unlike modern day submarines) surfacing was necessary to recharge batteries, vent the boat of fumes and recharge air tanks. Moreover U-boats were much faster on the surface than underwater as well as being less easily detected by sonar.

Accordingly, on the termination of the exercise, the Superintendent of Bawdsey Research Station, A.P. Rowe, announced that although the exercise had again demonstrated the technical feasibility of the radar system for detecting aircraft, its operational achievements still fell far short of requirements. He therefore proposed that a crash program of research into the operational - as opposed to the technical - aspects of the system should begin immediately. The term "*operational research*" was coined as a suitable description of this new branch of applied science. The first team was selected from amongst the scientists of the radar research group the same day.

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Thus the Operation Research started just before World War II in Britain with the establishment of teams of scientists to study the strategic and tactical problems involved in military operations. The objective was to find the most effective utilization of limited military resources by the use of quantitative techniques. Following the end of the war OR spread, although it spread in different ways in the UK and USA.

In 1951 a committee on Operations Research formed by the National Research Council of USA, and the first book on “Methods of Operations Research”, by Morse and Kimball, was published. In 1952 the Operations Research Society of America came into being.

Success of Operations Research in army attracted the attention of the industrial mangers who were seeking solutions to their complex business problems. Now a days, almost every organization in all countries has staff applying operations research, and the use of operations research in government has spread from military to wide variety of departments at all levels. The growth of operations research has not limited to the U.S.A. and U.K., it has reached many countries of the world.

India was one the few first countries who started using operations research. In India, Regional Research Laboratory located at Hyderabad was the first Operations Research unit established during 1949. At the same time another unit was set up in Defense Science Laboratory to solve the Stores, Purchase and Planning Problems. In 1953, Operations Research unit was established in Indian Statistical Institute, Calcutta, with the objective of using Operations Research methods in National Planning and Survey. In 1955, Operations Research Society of India was formed, which is one of the first members of International Federation of Operations Research societies. Today Operations Research is a popular subject in management institutes and schools of mathematics.

1.2.1 Stages of Development of Operations Research

The stages of development of O.R. are also known as phases and process of O.R, which has six important steps. These six steps are arranged in the following order:

Step I: Observe the problem environment

Step II: Analyze and define the problem

Step III: Develop a model

Step IV: Select appropriate data input

Step V: Provide a solution and test its reasonableness Step VI: Implement the solution

Step I: Observe the problem environment

The first step in the process of O.R. development is the problem environment observation. This step includes different activities; they are conferences, site visit, research, observations etc. These activities provide sufficient information to the O.R. specialists to formulate the problem.

Step II: Analyze and define the problem

This step is analyzing and defining the problem. In this step in addition to the problem definition the objectives, uses and limitations of O.R. study of the problem also defined. The outputs of this step are clear grasp of need for a solution and its nature understanding.

Step III: Develop a model

This step develops a model; a model is a representation of some abstract or real situation. The models are basically mathematical models, which describes systems, processes in the form of equations, formula/relationships. The different activities in this step are variables definition, formulating equations etc. The model is tested in the field under different environmental constraints and modified in order to work. Some times the model is modified to satisfy the management with the results.

Step IV: Select appropriate data input

A model works appropriately when there is appropriate data input. Hence, selecting appropriate input data is important step in the O.R. development stage or process. The activities in this step include internal/external data analysis, fact analysis, and collection of opinions and use of computer data banks. The objective of this step is to provide sufficient data input to operate and test the model developed in Step_III.

Step V: Provide a solution and test its reasonableness

This step is to get a solution with the help of model and input data. This solution is not implemented immediately, instead the solution is used to test the model and to find there is any limitations. Suppose if the solution is not reasonable or the behaviour of the model is not proper, the model is updated and modified at this stage. The output of this stage is the solution(s) that supports the current organizational objectives.

Step VI: Implement the solution

At this step the solution obtained from the previous step is implemented. The implementation of the solution involves mo many behavioural issues. Therefore, before implementation the implementation authority has to resolve the issues. A properly implemented solution results in quality of work and gains the support from the management.

The process, process activities, and process output are summarized in the following Table

Process Activities	Process	Process Output
Site visits, Conferenc Observations, Research	Step 1: Observe the probl environment	Sufficient information and support to proceed
Define: Use, Objectives, limitations	Step 2: Analyze and define the proble	Clear grasp of need for and nature of solution requested
Define interrelationships, Formulate equations, Use known O.R. Model ,	Step 3: Develop a Model	Models that works under stat environmental constraints

Search alternate Model		
Analyze: internal-external dat facts Collect options, Use computer data banks	Step 4: Select appropriate data input	Sufficient inputs to operate ar test model
Test the model find limitations update the model	Step 5: Provide a solution and test reasonableness	Solution(s) that support curre organizational goals

1.2.2 phases of operations research

1. **Problem Definition**
   - **Identify the Problem:** Clearly understand and define the problem to be solved.
   - **Set Objectives:** Determine the objectives and goals of the study.
   - **Constraints:** Identify the constraints and limitations within which the solution must operate.
2. **Data Collection**
   - **Gather Data:** Collect relevant data that will be used in the analysis.
   - **Data Validation:** Ensure the accuracy and reliability of the data collected.
3. **Model Construction**
   - **Formulate Models:** Develop mathematical models that represent the system or problem.
   - **Assumptions:** Make necessary assumptions to simplify the model while retaining its essential characteristics.

4. **Model Solution**

- **Solve Models:** Use appropriate algorithms and computational techniques to solve the models.
- **Validation:** Validate the model by comparing its outputs with real-world data to ensure its accuracy.

5. **Implementation**

- **Implement Solutions:** Put the chosen solution into practice.
- **Monitor and Control:** Continuously monitor the implemented solution and make adjustments as necessary.

6. **Review and Feedback**

- **Evaluate Performance:** Assess the performance of the implemented solution.
- **Feedback Loop:** Use the feedback to refine the model and make further improvements.

Let Us Sum Up

In studying Operations Research (OR) and Linear Programming (LP), learners gain a comprehensive understanding of how to apply mathematical and analytical techniques to optimize decision-making processes in complex systems. Operations Research introduces the foundational concepts and methodologies for addressing real-world problems where resources are limited and must be allocated efficiently.

1.1 Check Your Progress

1.What is the primary goal of Operations Research (OR)? a) To develop new mathematical theories

b) To make better decisions through advanced analytical methods

c) To understand natural phenomena

d) To increase computational complexity

2.Which of the following is NOT a typical application of Linear Programming? a) Maximizing profit

b) Minimizing cost

c) Scheduling production

d) Predicting weather patterns

3.In a Linear Programming problem, what does the objective function represent? a) The constraints of the problem

b) The feasible region

c) The goal to be optimized

d) The decision variables

4.What method is commonly used to solve Linear Programming problems? a) Monte Carlo simulation

b) Simplex method

c) Gradient descent

d) Euler's method

5.Which of the following conditions must be true for a Linear Programming problem? a) The objective function and constraints must be nonlinear

b) The decision variables can be continuous or discrete

c) The constraints must form a convex feasible region

d) There must be no more than three decision variables

1.3 APPROACHES TO OPERATIONS RESEARCH

1. **Optimization**
    - **Linear Programming (LP):** Used for problems that can be expressed as linear relationships.
    - **Integer Programming (IP):** Deals with optimization problems where some or all variables are restricted to integer values.
    - **Nonlinear Programming (NLP):** Used when the relationships are nonlinear.
    - **Dynamic Programming:** Solves problems by breaking them down into simpler subproblems.
2. **Simulation**
    - **Monte Carlo Simulation:** Uses random sampling to estimate complex mathematical or physical systems.

- **Discrete-Event Simulation:** Models the operation of a system as a discrete sequence of events in time.

3. **Statistical Analysis**
    - **Regression Analysis:** Analyzes relationships among variables.
    - **Time Series Analysis:** Analyzes data points collected or recorded at specific time intervals.
4. **Queuing Theory**
    - **Modeling Waiting Lines:** Analyzes and models the behavior of queues in systems such as customer service, manufacturing, and telecommunications.
5. **Decision Analysis**
    - **Decision Trees:** Used to visualize decisions and their possible consequences.
    - **Utility Theory:** Helps in making decisions under uncertainty by considering the utility of outcomes.
6. **Game Theory**
    - **Strategic Decision Making:** Analyzes competitive situations where the outcome depends on the actions of multiple agents.
7. **Network Models**
    - **Shortest Path:** Finds the shortest path in a network.
    - **Maximum Flow:** Determines the maximum possible flow in a network.
    - **Project Scheduling:** Techniques like PERT and CPM are used for planning and managing projects.

1.3.1 Integrated Approaches

1. **Hybrid Methods**
    - **Combining Techniques:** Use a combination of methods (e.g., simulation with optimization) to tackle complex problems more effectively.

2. **Systems Analysis**

   - **Holistic View:** Considers the entire system, including interactions between its components, to identify and solve problems.

1.5 TOOLS AND TECHNIQUES

Operations Research uses any suitable tools or techniques available. The common frequently used tools/techniques are mathematical procedures, cost analysis, electronic computation. However, operations researchers given special importance to the development and the use of techniques like linear programming, game theory, decision theory, queuing theory, inventory models and simulation. In addition to the above techniques, some other common tools are non-linear programming, integer programming, dynamic programming, sequencing theory, Markov process, network scheduling (PERT/CPM), symbolic Model, information theory, and value theory. There is many other Operations Research tools/techniques also exists. The brief explanations of some of the above techniques/tools are as follows:

Linear Programming

This is a constrained optimization technique, which optimize some criterion within some constraints. In Linear programming the objective function (profit, loss or return on investment) and constraints are linear. There are different methods available to solve linear programming.

Game Theory

This is used for making decisions under conflicting situations where there are one or more players/opponents. In this the motive of the players are dichotomized. The success of one player tends to be at the cost of other players and hence they are in conflict.

Decision Theory

Decision theory is concerned with making decisions under conditions of complete certainty about the future outcomes and under conditions such that we can make some probability about what will happen in future.

Queuing Theory

This is used in situations where the queue is formed (for example customers waiting for service, aircrafts waiting for landing, jobs waiting for processing in the computer system, etc). The objective here is minimizing the cost of waiting without increasing the cost of servicing.

Inventory Models

Inventory model make a decisions that minimize total inventory cost. This model successfully reduces the total cost of purchasing, carrying, and out of stock inventory.Simulation:

Simulation

Is a procedure that studies a problem by creating a model of the process involved in the problem and then through a series of organized trials and error solutions attempt to determine the best solution. Some times this is a difficult/time consuming procedure. Simulation is used when actual experimentation is not feasible or solution of model is not possible.

Non-linear Programming:

This is used when the objective function and the constraints are not linear in nature. Linear relationships may be applied to approximate non-linear constraints but limited to some range, because approximation becomes poorer as the range is extended. Thus, the non-linear programming is used to determine the approximation in which a solution lies and then the solution is obtained using linear methods.

Dynamic Programming:

Dynamic programming is a method of analyzing multistage decision processes. In this each elementary decision depends on those preceding decisions and as well as external factors.

Integer Programming:

If one or more variables of the problem take integral values only then dynamic programming method is used. For example number or motor in an organization, number of passenger in an aircraft, number of generators in a power generating plant, etc.

Markov Process:

Markov process permits to predict changes over time information about the behavior of a system is known. This is used in decision making in situations where the various states are defined. The probability from one state to another state is known and depends on the current state and is independent of how we have arrived at that particular state.

Network Scheduling:

This technique is used extensively to plan, schedule, and monitor large projects (for example computer system installation, R & D design, construction, maintenance, etc.). The aim of this technique is minimize trouble spots (such as delays, interruption, production bottlenecks, etc.) by identifying the critical factors. The different activities and their relationships of the entire project are represented diagrammatically with the help of networks and arrows, which is used for identifying critical activities and path. There are two main types of technique in network scheduling, they are:

Program Evaluation and Review Technique (PERT) – is used when activities time is not known accurately/ only probabilistic estimate of time is available.

Critical Path Method (CPM) – is used when activities time is know accurately.

Information Theory

This analytical process is transferred from the electrical communication field to O.R. field. The objective of this theory is to evaluate the effectiveness of flow of information with a given system. This is used mainly in communication networks but also has indirect influence in simulating the examination of business organizational structure with a view of enhancing flow of information.

1.6 APPLICATIONS OF OPERATIONS RESEARCH

Today, almost all fields of business and government utilizing the benefits of Operations Research. There are voluminous of applications of Operations Research. Although it is not feasible to cover all applications of O.R. in brief. The following are the abbreviated set of typical operations research applications to show how widely these techniques are used today:

Today, almost all fields of business and government utilizing the benefits of Operations Research. There are voluminous of applications of Operations Research. Although it is not feasible to cover all applications of O.R. in brief. The following are the abbreviated set of typical operations research applications to show how widely these techniques are used today:

Accounting:

Assigning audit teams effectively Credit policy analysis

Cash flow planning Developing standard costs

Establishing costs for byproducts Planning of delinquent account strategy

Construction:

Project scheduling, monitoring and control Determination of proper work force Deployment of work force Allocation of resources to projects

Facilities Planning:

Factory location and size decision Estimation of number of facilities required Hospital planning International logistic system design Transportation loading and unloading Warehouse location decision

Finance:

Building cash management models Allocating capital among various alternatives Building financial planning models Investment analysis Portfolio analysis Dividend policy making

Manufacturing:

Inventory control Marketing balance projection Production scheduling Production

smoothing

Marketing:

Advertising budget allocation Product introduction timing Selection of Product mix Deciding most effective packaging alternative

Organizational Behavior / Human Resources:

Personnel planning Recruitment of employees Skill balancing Training program scheduling Designing organizational structure more effectively

Purchasing:

Optimal buying Optimal reordering Materials transfer

Research and Development:

R & D Projects control R & D Budget allocation, Planning of Product introduction

1.7 Limitations of Operations Research

Operations Research has number of applications; similarly it also has certain limitations. These limitations are mostly related to the model building and money and time factors problems involved in its application. Some of them are as given below:

Distance between O.R. specialist and Manager

Operations Researchers job needs a mathematician or statistician, who might not be aware of the business problems. Similarly, a manager is unable to understand the complex nature of Operations Research. Thus there is a big gap between the two personnel.

Magnitude of Calculations

The aim of the O.R. is to find out optimal solution taking into consideration all the factors. In this modern world these factors are enormous and expressing them in quantitative model and establishing relationships among these require voluminous calculations, which can be handled only by machines.

Money and Time Costs

The basic data are subjected to frequent changes, incorporating these changes into the operations research models is very expensive. However, a fairly good solution at present may be more desirable than a perfect operations research solution available in future or after some time.

Non-quantifiable Factors

When all the factors related to a problem can be quantifiable only then operations research provides solution otherwise not. The non-quantifiable factors are not incorporated in O.R. models. Importantly O.R. models do not take into account emotional factors or qualitative factors.

Implementation

Once the decision has been taken it should be implemented. The implementation of decisions is a delicate task. This task must take into account the complexities of human relations and behavior and in some times only the psychological factors.

1.8 SIMPLEX METHOD

The simplex method is an algorithm used for solving linear programming problems. Linear programming involves optimizing a linear objective function, subject to a set of linear inequality or equality constraints. The simplex method, developed by George Dantzig in 1947, is one of the most widely used algorithms for this purpose. Here is a detailed explanation of the simplex method:

Meaning and Purpose of the Simplex Method

1. **Optimization Technique:**

- The simplex method is designed to find the maximum or minimum value of a linear objective function.
- It does this by navigating the vertices (or "corners") of the feasible region defined by the linear constraints.

2. **Linear Programming Problem Formulation:**

   - **Objective Function:** The function to be maximized or minimized, often represented as Z=c1x1+c2x2+⋯+cnxnZ = c_1x_1 + c_2x_2 + \cdots + c_nx_nZ=c1x1+c2x2+⋯+cnxn.
   - **Constraints:** Linear inequalities or equalities that define the feasible region. These can be written in the form a11x1+a12x2+⋯+a1nxn≤b1a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \leq b_1a11x1+a12x2+⋯+a1nxn≤b1, and so on.
   - **Non-negativity Restrictions:** The decision variables x1,x2,…,xnx_1, x_2, \ldots, x_nx1,x2,…,xn must be non-negative, i.e., xi≥0x_i \geq 0xi≥0 for all iii.

3. **Steps of the Simplex Method:**

   - **Initialization:** Start with an initial basic feasible solution, typically at one of the vertices of the feasible region.
   - **Iteration:** Move from one vertex of the feasible region to an adjacent vertex with a better objective function value. This is done by pivoting, a process involving entering and leaving variables.
   - **Optimality Check:** Check if the current solution satisfies the optimality conditions. If it does, the algorithm terminates; otherwise, continue iterating.

4. Geometric Interpretation:

- The feasible region in a linear programming problem is a convex polyhedron (in two dimensions, it is a polygon; in three dimensions, a polyhedron, and so on).
- The simplex method traverses the edges of this polyhedron from one vertex to another, always improving or maintaining the objective function value, until the optimal vertex is reached.

1.8.1 Steps in computation

The Simplex method is an approach to solving linear programming models by hand using slack variables, tableaus, and pivot variables as a means to finding the optimal solution of an optimization problem. A linear program is a method of achieving the best outcome given a maximum or minimum equation with linear constraints. Most linear programs can be solved using an online solver such as MatLab, but the Simplex method is a technique for solving linear programs by hand. To solve a linear programming model using the Simplex method the following steps are necessary:

- Standard form
- Introducing slack variables
- Creating the tableau
- Pivot variables
- Creating a new tableau
- Checking for optimality
- Identify optimal values

This document breaks down the Simplex method into the above steps and follows the example linear programming model shown below throughout the entire document to find the optimal solution.

$$Minimize : -z = -8x_1 - 10x_2 - 7x_3$$
$$s.t. : x_1 + 3x_2 + 2x_3 \leq 10$$
$$-x_1 - 5x_2 - x_3 \geq -8$$
$$x_1, x_2, x_3 \geq 0$$

Step 1: Standard Form

Standard form is the baseline format for all linear programs before solving for the optimal solution and has three requirements: (1) must be a maximization problem, (2) all linear constraints must be in a less-than-or-equal-to inequality, (3) all variables are non-negative. These requirements can always be satisfied by transforming any given linear program using basic algebra and substitution. Standard form is necessary because it creates an ideal starting point for solving the Simplex method as efficiently as possible as well as other methods of solving optimization problems.

To transform a minimization linear program model into a maximization linear program model,

$$-1 \times (-z = -8x_1 - 10x_2 - 7x_3)$$
$$z = 8x_1 + 10x_2 + 7x_3$$
$$Maximize: z = 8x_1 + 10x_2 + 7x_3$$

simply multiply both the left and the right sides of the objective function by -1.

Transforming linear constraints from a greater-than-or-equal-to inequality to a less-than-or-equal-to inequality can be done similarly as what was done to the objective function. By multiplying by -1 on both sides, the inequality can be changed to less-than-or-equal-to.

$$-1 \times (-x_1 - 5x_2 - x_3 \geq -8)$$
$$x_1 + 5x_2 + x_3 \leq 8$$

Once the model is in standard form, the slack variables can be added as shown in Step 2 of the Simplex method.

Step 2: Determine Slack Variables

Slack variables are additional variables that are introduced into the linear constraints of a linear program to transform them from inequality constraints to equality constraints. If the model is in standard form, the slack variables will always have a +1 coefficient. Slack variables are needed in the constraints to transform them into solvable equalities with one definite answer.

$$x_1 + 3x_2 + 2x_3 + \mathbf{s_1} = 10$$
$$x_1 + 5x_2 + x_3 + \mathbf{s_2} = 8$$
$$x_1, x_2, x_3, \mathbf{s_1}, \mathbf{s_2} \geq 0$$

After the slack variables are introduced, the tableau can be set up to check for optimality as described in Step 3.

Step 3: Setting up the Tableau

A Simplex tableau is used to perform row operations on the linear programming model as well as to check a solution for optimality. The tableau consists of the coefficient corresponding to the linear constraint variables and the coefficients of the objective function. In the tableau below, the bolded top row of the tableau states what each column represents. The following two rows represent the linear constraint variable coefficients from the linear programming model, and the

$$Maximize: z = 8x_1 + 10x_2 + 7x_3$$
$$s.t.: x_1 + 3x_2 + 2x_3 + s_1 = 10$$
$$x_1 + 5x_2 + x_3 + s_2 = 8$$

last row represents the objective function variable coefficients.

Once the tableau has been completed, the model can be checked for an optimal solution as shown in Step 4.

x1	x2	x3	s1	s2	z	b
1	3	2	1	0	0	10
1	5	1	0	1	0	8
-8	-10	-7	0	0	1	0

Step 4: Check Optimality

The optimal solution of a maximization linear programming model are the values assigned to the variables in the objective function to give the largest zeta value. The optimal solution would exist on the corner points of the graph of the entire model. To check optimality using the tableau, all values in the last row must contain values greater than or equal to zero. If a value is less than zero, it means that variable has not reached its optimal value. As seen in the previous tableau, three negative values exists in the bottom row indicating that this solution is not optimal. If a tableau is not optimal, the next step is to identify the pivot variable to base a new tableau on, as described in Step 5.

Step 5: Identify Pivot Variable

The pivot variable is used in row operations to identify which variable will become the unit value and is a key factor in the conversion of the unit value. The pivot variable can be identified by looking at the bottom row of the tableau and the indicator. Assuming that the solution is not optimal, pick the smallest negative value in the bottom row. One of the values lying in the column of this value will be the pivot variable. To find the indicator, divide the beta values of the linear constraints by their corresponding values from the column containing the possible pivot variable. The intersection of the row with the smallest non-negative indicator and the smallest negative value in the bottom row will become the pivot variable.

In the example shown below, -10 is the smallest negative in the last row. This will designate the x_2 column to contain the pivot variable. Solving for the indicator gives us a value of $\frac{10}{3}$for the first constraint, and a value of $\frac{8}{5}$for the second constraint. Due to $\frac{8}{5}$ being the smallest non-negative indicator, the pivot value will be in the second row and have a value of 5.

x1	x2	x3	s1	s2	z	b	Indicator
1	3	2	1	0	0	10	10/3
1	(5)	1	0	1	0	8	8/5
-8	-10	-7	0	0	1	0	
	↑ Smallest Value						

Now that the new pivot variable has been identified, the new tableau can be created in Step 6 to optimize the variable and find the new possible optimal solution.

Step 6: Create the New Tableau

The new tableau will be used to identify a new possible optimal solution. Now that the pivot variable has been identified in Step 5, row operations can be performed to optimize the pivot variable while keeping the rest of the tableau equivalent.

I. To optimize the pivot variable, it will need to be transformed into a unit value (value of 1). To transform the value, multiply the row containing the pivot variable by the reciprocal of the pivot value. In the example below, the pivot variable is originally 5, so multiply the entire row by $\frac{1}{5}$.

x1	x2	x3	s1	s2	z	b	
1/5	(1)	1/5	0	1/5	0	8/5	← Pivot row

x1	x2	x3	s1	s2	z	b	
	0						
1/5	(1)	1/5	0	1/5	0	8/5	← Pivot row
	0						
	↑ Pivot Column						

II. After the unit value has been determined, the other values in the column containing the unit value will become zero. This is because the x_2 in the second constraint is being optimized, which requires x_2 in the other equations to be zero. iii. In order to keep the tableau equivalent, the other variables not contained in the pivot column or pivot row must be calculated by using the new pivot values. For each new value, multiply the negative of the value in the old pivot column by the value in the new pivot row that corresponds to the value being calculated. Then add this to the old value from the old tableau to produce the new value for the new tableau. This step can be condensed into the equation on the next page:

New tableau value = (Negative value in old tableau pivot column) x (value in new tableau pivot row) + (Old tableau value)

Old Tableau:

x1	x2	x3	s1	s2	z	b
1	3	2	1	0	0	10
1	(5)	1	0	1	0	8
-8	-10	-7	0	0	1	0
	↑ Old pivot column					

New Tableau:

x1	x2	x3	s1	s2	z	b
2/5	0	7/5	1	-3/5	0	26/5
1/5	(1)	1/5	0	1/5	0	8/5 ← New pivot row
-6	0	-5	0	2	1	16

Numerical examples are provided below to help explain this concept a little better.

Numerical examples:

i. To find the s_2 value in row 1:

New tableau value = (Negative value in old tableau pivot column) * (value in new tableau pivot row) + (Old tableau value)

New tableau value = (-3) * ($\frac{1}{5}$) + 0 = -$\frac{3}{5}$

I. To find the x_1 variable in row 3:

New tableau value = (Negative value in old tableau pivot column) * (value in new tableau pivot row) + (Old tableau value)

New value = (10) * ($\frac{1}{5}$) + -8 = -6

Once the new tableau has been completed, the model can be checked for an optimal solution.

Step 7: Check Optimality

As explained in Step 4, the optimal solution of a maximization linear programming model are the values assigned to the variables in the objective function to give the largest zeta value. Optimality will need to be checked after each new tableau to see if a new pivot variable needs to be identified. A solution is considered optimal if all values in the bottom row are greater than or equal to zero. If all values are greater than or equal to zero, the solution is considered optimal and Steps 8 through 11 can be ignored. If negative values exist, the solution is still not optimal and a new pivot point will need to be determined which is demonstrated in Step 8.

Step 8: Identify New Pivot Variable

If the solution has been identified as not optimal, a new pivot variable will need to be determined. The pivot variable was introduced in Step 5 and is used in row operations to identify which variable will become the unit value and is a key factor in the conversion of the unit value. The pivot variable can be identified by the intersection of the row with the smallest non-negative indicator and the smallest negative value in the bottom row.

x1	x2	x3	s1	s2	z	b	Indicator
2/5	0	7/5	1	−3/5	0	26/5	(26/5) / (2/5) = 13
(1/5)	1	1	0	1/5	0	8/5	(8/5) / (1/5) = 8
-6	0	-5	0	2	1	0	

↑ Smallest Value

With the new pivot variable identified, the new tableau can be created in Step 9.

Step 9: Create New Tableau

After the new pivot variable has been identified, a new tableau will need to be created. Introduced in Step 6, the tableau is used to optimize the pivot variable while keeping the rest of the tableau equivalent.

I. Make the pivot variable 1 by multiplying the row containing the pivot variable by the reciprocal of the pivot value. In the tableau below, the pivot value was $\frac{1}{5}$, so everything is multiplied by 5.

x1	x2	x3	s1	s2	z	b
(1)	5	1	0	1	0	8

II. Next, make the other values in the column of the pivot variable zero. This is done by taking the negative of the old value in the pivot column and multiplying it by the new value in the pivot row. That value is then added to the old value that is being replaced.

x1	x2	x3	s1	s2	z	b
0	-2	1	1	-1	0	2
(1)	5	1	0	1	0	8
0	30	1	0	8	1	64

Step 10: Check Optimality

Using the new tableau, check for optimality. Explained in Step 4, an optimal solution appears when all values in the bottom row are greater than or equal to zero. If all values are greater than or equal to zero, skip to Step 12 because optimality has been reached. If negative values still exist, repeat steps 8 and 9 until an optimal solution is obtained.

Step 11: Identify Optimal Values

Once the tableau is proven optimal the optimal values can be identified. These can be found by distinguishing the basic and non-basic variables. A basic variable can be classified to have a single 1 value in its column and the rest be all zeros. If a variable does not meet this criteria, it is considered non-basic. If a variable is non-basic it means the optimal solution of that variable is zero. If a variable is basic, the row that contains the 1 value will correspond to the beta value. The beta value will represent the optimal solution for the given variable.

x1	x2	x3	s1	s2	z	b
0	-2	1	1	-1	0	2
1	5	1	0	1	0	8
0	30	1	0	8	1	64

Basic variables: x_1, s_1, z

Non-basic variables: x_2, x_3, s_2

For the variable x_1, the 1 is found in the second row. This shows that the optimal x_1 value is found in the second row of the beta values, which is 8.

Variable s_1 has a 1 value in the first row, showing the optimal value to be 2 from the beta column. Due to s_1 being a slack variable, it is not actually included in the optimal solution since the variable is not contained in the objective function.

The zeta variable has a 1 in the last row. This shows that the maximum objective value will be 64 from the beta column.

The final solution shows each of the variables having values of:

x_1	= 8	s_1	= 2
x_2	= 0	s_2	= 0
x_3	= 0	z	= 64

The maximum optimal value is 64 and found at (8, 0, 0) of the objective function.

Conclusion

The Simplex method is an approach for determining the optimal value of a linear program by hand. The method produces an optimal solution to satisfy the given constraints and produce a maximum zeta value. To use the Simplex method, a given linear programming model needs to be in standard form, where slack variables can then be introduced. Using the tableau and pivot variables, an optimal solution can be reached. From the example worked throughout this document, it can be determined that the optimal objective value is 64 and can be found when x_1=8, x_2=0, and x_3=0.

1.9 GRAPHICAL METHOD

The graphical method is a technique used to solve linear programming problems with two decision variables. This method provides a visual representation of the feasible region and the objective function, making it easier to understand the problem and identify the optimal solution. Graphical Method: Owing to the importance of linear programming models in various industries, many types of algorithms have been developed over the years to solve them. Some famous mentions include the Simplex method, the Hungarian approach, and others. Here we are going to concentrate on one of the most basic methods to handle a linear programming problem i.e. the graphical method.

In principle, this method works for almost all different types of problems but gets more and more difficult to solve when the number of decision variables and the constraints increases

Here's an in-depth look at the graphical method:

Meaning and Purpose of the Graphical Method

1. Optimization Technique:
   - The graphical method is used to find the maximum or minimum value of a linear objective function.
   - It involves plotting the constraints and objective function on a two-dimensional graph to visualize the feasible region and determine the optimal solution.
2. Problem Formulation:
   - Objective Function: The function to be optimized, usually in the form Z=c1x1+c2x2Z = c_1x_1 + c_2x_2Z=c1x1+c2x2.
   - Constraints: Linear inequalities that define the feasible region, such as a1x1+b1x2≤c1a_1x_1 + b_1x_2 \leq c_1a1x1+b1x2≤c1 and a2x1+b2x2≤c2a_2x_1 + b_2x_2 \leq c_2a2x1+b2x2≤c2.
   - Non-negativity Restrictions: The decision variables x1x_1x1 and x2x_2x2 must be non-negative, i.e., x1≥0x_1 \geq 0x1≥0 and x2≥0x_2 \geq 0x2≥0.

1.9.1 Steps of the Algorithm

Step 1: Formulate the LP (Linear programming) problem

We have already understood the mathematical formulation of an LP problem in a previous section.

Step 2: Construct a graph and plot the constraint lines

The graph must be constructed in 'n' dimensions, where 'n' is the number of decision variables. This should give you an idea about the complexity of this step if the number of decision variables increases.One must know that one cannot imagine more than 3-dimensions anyway! The constraint lines can be constructed by joining the horizontal and vertical intercepts found from each constraint equation.

Step 3: Determine the valid side of each constraint line

This is used to determine the domain of the available space, which can result in a feasible solution. How to check? A simple method is to put the coordinates of the origin (0,0) in the problem and determine whether the objective function takes on a physical solution or not. If yes, then the side of the constraint lines on which the origin lies is the valid side. Otherwise it lies on te opposite one.

Step 4: Identify the feasible solution region

The feasible solution region on the graph is the one which is satisfied by all the constraints. It could be viewed as the intersection of the valid regions of each constraint line as well. Choosing any point in this area would result in a valid solution for our objective function.

Step 5: Plot the objective function on the graph

It will clearly be a straight line since we are dealing with linear equations here. One must be sure to draw it differently from the constraint lines to avoid confusion. Choose the constant value in the equation of the objective function randomly, just to make it clearly distinguishable.

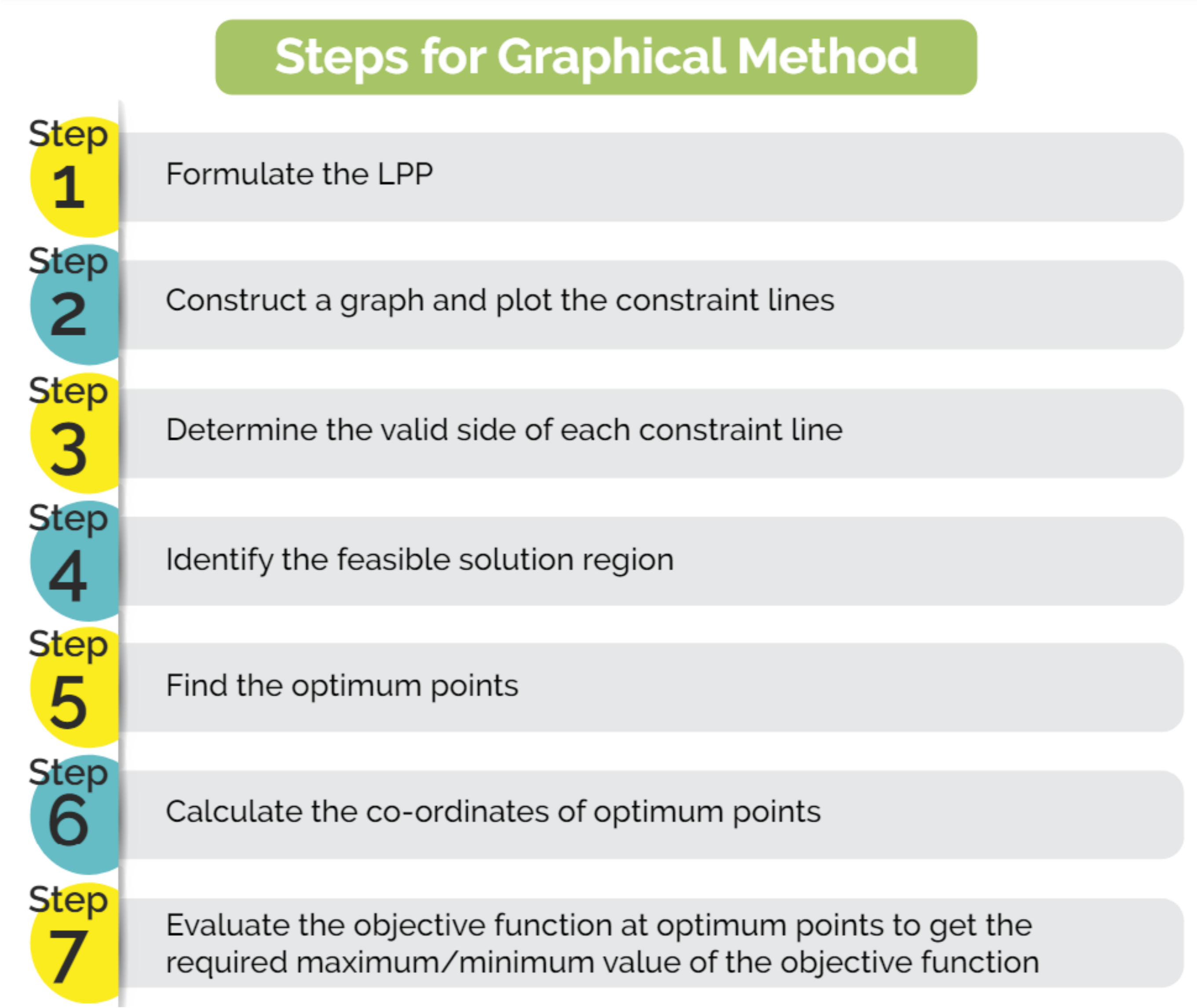
Step 6: Find the optimum point

Optimum Points

An optimum point always lies on one of the corners of the feasible region. How to find it? Place a ruler on the graph sheet, parallel to the objective function. Be sure to keep the orientation of this ruler fixed in space. We only need the direction of the straight line of the objective function. Now begin from the far corner of the graph and tend to slide it towards the origin.

If the goal is to minimize the objective function, find the point of contact of the ruler with the feasible region, which is the closest to the origin. This is the optimum point for minimizing the function.

If the goal is to maximize the objective function, find the point of contact of the ruler with the feasible region, which is the farthest from the origin. This is the optimum point for maximizing the function.



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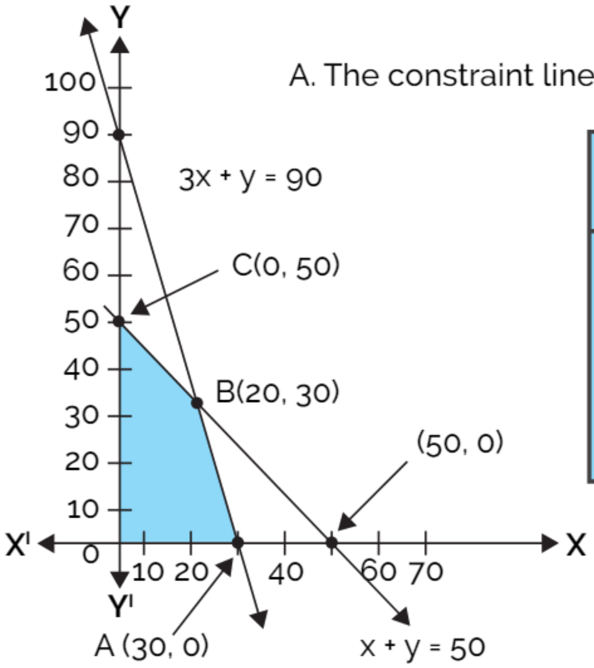
Solved Example

Q. Maximize and minimize z = 4x + y subject to:- $x + y \le 50$

$3x + y \le 90$

$x \ge 0, y \ge 0$

A. The constraint lines are x + y = 50, 3x + y = 90, x = 0, y = 0



Corner Point	Corresponding value of Z
(0, 0)	0
(30, 0)	120
(20, 30)	110
(0, 50)	50

Hence, maximum value of Z is 120 at the point (30, 0) and the minimum value of z is 0 at the point (0, 0).

Let Us Sum-up

The study of Operations Research (OR) equips learners with a diverse set of tools and techniques designed to optimize decision-making processes in complex systems. These tools include mathematical modeling, statistical analysis, and algorithmic approaches, which are applied to solve real-world problems in various domains such as logistics, manufacturing, finance, and healthcare.

Check Your Progress

1.Which of the following is a key technique in Operations Research used for optimizing a linear objective function subject to linear constraints? a) Integer Programming

b) Linear Programming

c) Simulation

d) Network Analysis

2.Which Operations Research tool is used for optimizing flows and connectivity in systems such as supply chains and transportation networks? a) Queueing Theory

b) Decision Trees

c) Network Models

d) Monte Carlo Simulation

3.What technique in Operations Research is specifically designed to handle optimization problems where some or all decision variables are restricted to integer values? a) Linear Programming

b) Simulation

c) Integer Programming

d) Queueing Theory

4.Which technique is used in Operations Research to model and analyze the behavior of systems under uncertainty? a) Simulation

b) Integer Programming

c) Linear Programming

d) Network Models

5.Which Operations Research method involves the study of waiting lines and service systems to improve efficiency and reduce delays? a) Decision Trees

b) Queueing Theory

c) Multi-Criteria Decision Making

d) Network Analysis

UNIT SUMMARY

Operations research (OR) involves the application of analytical methods to aid decision-making and problem-solving in complex systems. Key areas of learning in OR include linear programming, which optimizes resource allocation; network models, used for logistics and supply chain management; and integer programming, for decisions requiring discrete choices. Additionally, OR covers queuing theory for analyzing waiting lines, simulation techniques for modeling uncertain environments, and inventory models to manage stock levels efficiently. The focus is on developing mathematical models, employing computational tools, and utilizing statistical analysis to improve operational efficiency, reduce costs, and enhance productivity across various industries.

GLOSSARY

1. **Linear Programming (LP)**: A mathematical method used to determine the best possible outcome, such as maximum profit or lowest cost, in a given mathematical model whose requirements are represented by linear relationships.
2. **Queuing Theory**: The study of waiting lines or queues. It uses mathematical models to analyze the behavior and performance of queuing systems, helping in the design and management of systems like customer service, telecommunications, and traffic flow.
3. **Simulation**: A technique that models the operation of a system as it evolves over time. It allows the study of complex systems by creating a computer-based model to observe its behavior under different scenarios and conditions.
4. **Integer Programming (IP)**: A type of mathematical optimization where some or all of the decision variables are required to be integers. It is used in situations where the decisions are discrete, such as scheduling, planning, and resource allocation.
5. **Network Models**: Mathematical representations of networks used to optimize various logistical and transportation problems. Examples include the shortest path problem, the traveling salesman problem, and the maximum flow problem, which are essential for efficient routing and distribution in supply chain management.

Self-Assessment Questions

1. Define Operations Research.
2. Describe the relationship between the manager and O.R. specialist.
3. Explain the various steps in the O.R. development process.
4. Discuss the applications of O.R.
5. Discuss the limitation of O.R.
6. Describe different techniques of O.R.

Exercise

7. Discuss few areas of O.R. applications in your organization or organization you are familiar with.

8. Maximize Z=4x1+3x2Z = 4x_1 + 3x_2Z=4x1+3x2 using simplex method

   Subject to:

   2x1+x2≤102x_1 + x_2 \leq 102x1+x2≤10

   x1+2x2≤8x_1 + 2x_2 \leq 8x1+2x2≤8

   x1,x2≥0x_1, x_2 \geq 0x1,x2≥0

9. Maximize Z=4x1+3x2Z = 4x_1 + 3x_2Z=4x1+3x2 using simplex method

   Subject to:

   2x1+x2≤102x_1 + x_2 \leq 102x1+x2≤10

   x1+2x2≤8x_1 + 2x_2 \leq 8x1+2x2≤8

   x1,x2≥0x_1, x_2 \geq 0x1,x2≥0

10. Maximize Z=2x1+3x2Z = 2x_1 + 3x_2Z=2x1+3x2 using simplex method

   Subject to:

   3x1+4x2≤243x_1 + 4x_2 \leq 243x1+4x2≤24

   x1+2x2≤10x_1 + 2x_2 \leq 10x1+2x2≤10

   x1,x2≥0x_1, x_2 \geq 0x1,x2≥0

11. Maximize Z=3x1+5x2Z = 3x_1 + 5x_2Z=3x1+5x2 using simplex method

   Subject to:

   x1+2x2≤10x_1 + 2x_2 \leq 10x1+2x2≤10

   2x1+x2≤102x_1 + x_2 \leq 102x1+x2≤10

   x1,x2≥0x_1, x_2 \geq 0x1,x2≥0

12. Maximize Z=3x1+4x2Z = 3x_1 + 4x_2Z=3x1+4x2 using graphical method

Subject to:

x1≥0x_1 \geq 0x1≥0

x2≥0x_2 \geq 0x2≥0

2x1+x2≤62x_1 + x_2 \leq 62x1+x2≤6

x1+2x2≤8x_1 + 2x_2 \leq 8x1+2x2≤8

13. Minimize Z=5x1+4x2Z = 5x_1 + 4x_2Z=5x1+4x2 using graphical method

Subject to:

x1≥0x_1 \geq 0x1≥0

x2≥0x_2 \geq 0x2≥0

x1+x2≥5x_1 + x_2 \geq 5x1+x2≥5

2x1+3x2≥122x_1 + 3x_2 \geq 122x1+3x2≥12

14. Maximize Z=2x1+3x2Z = 2x_1 + 3x_2Z=2x1+3x2 using graphical method

Subject to:

x1≥0x_1 \geq 0x1≥0

x2≥0x_2 \geq 0x2≥0

x1+2x2≤4x_1 + 2x_2 \leq 4x1+2x2≤4

2x1+x2≤62x_1 + x_2 \leq 62x1+x2≤6

15. Maximize Z=4x1+3x2Z = 4x_1 + 3x_2Z=4x1+3x2

Subject to:

- x1≥0x_1 \geq 0x1≥0
- x2≥0x_2 \geq 0x2≥0
- x1+2x2≤8x_1 + 2x_2 \leq 8x1+2x2≤8
- 2x1+x2≤102x_1 + x_2 \leq 102x1+x2≤10

Check Your Progress Answers

1.1

b) To make better decisions through advanced analytical methods

d) Predicting weather patterns

c) The goal to be optimized

b) Simplex method

c) The constraints must form a convex feasible region

1.2

b) Linear Programming

c) Network Models

c) Integer Programming

a) Simulation

b) Queueing Theory

UNIT II

Unit Introduction

The Transportation and Assignment problems are fundamental topics within the field of Operations Research that deal with optimizing logistics and resource allocation. The Transportation Problem focuses on determining the most cost-effective way to distribute a product from several suppliers to several consumers while satisfying supply and demand constraints. This involves minimizing the total transportation cost, taking into account the shipping costs between each supplier-consumer pair. The problem can be solved using specialized methods such as the Northwest Corner Rule, the Least Cost Method, and the Vogel Approximation Method, followed by optimization techniques like the MODI Method to achieve the best solution.

2.1 TRANSPORTATION AND ASSIGNMENT PROBLEMS

The transportation model is a special class of linear programs. It received this name because many of its applications involve determining how to optimally transport goods. However, some of its important applications (eg production scheduling) actually have nothing to do with transportation.

The second type of problem is assignment problem. It involves such applications as assigning people to tasks. Although its applications appear to be quite different from those for the transportation, we shall see the assignment problem can be viewed as a special type of transportation problem.

Application of the transportation and assignment problem tend to require a very large number of constraints and variables, so straightforward computer applications of simplex method may require an exorbitant computational effort. Fortunately, a key characteristic of these problems is that most of the aij coefficient in the constraints are zeros, and the relatively few non-zero coefficient appear in a distinctive pattern. As a result, it has been possible to develop special streamlined algorithms that achieve dramatic computational savings by exploiting this special structure of the problem. Therefore, it is important to become sufficiently familiar with these special types of problems that you can recognize them when they arise and apply the proper computational procedure.

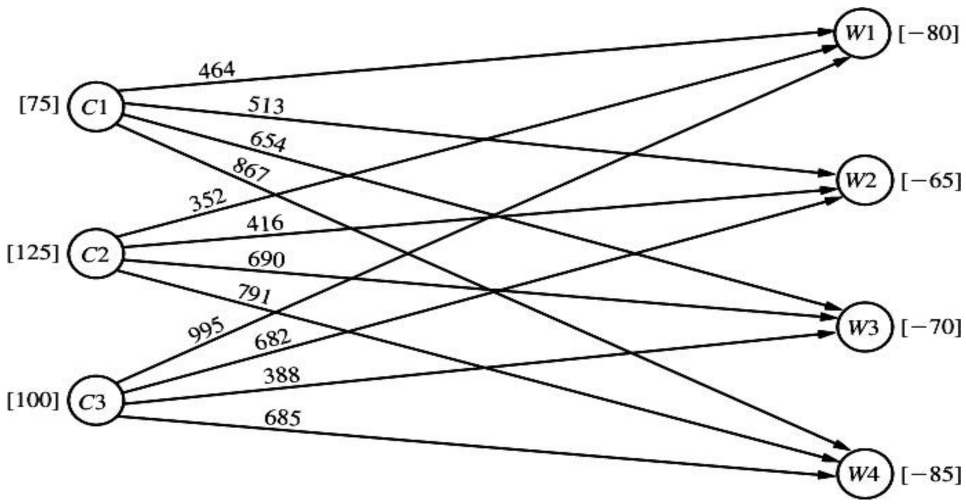
As pointed out above that many applications of TP deals with transportation of a commodity from source to destination. Therefore I will use prototype example to explain the TP.

Example- Suppose XYZ Pvt Ld produce good X. The production take place at 3 place- S1, S2 and S3. From these production places, company supplies the X to its warehouses, which are located near its demand centre. The shipping cost or transportation cost from different production place to warehouses is given table 1

Si represents the production place(sources) and Wi represents the warehouses (destinations). We can represent the above problem with network diagram as following

	Shpping cost per unit				
	W1	W2	W3	W4	Output
S1	464	513	654	867	75
S2	352	416	690	791	125
S3	995	682	388	685	100
Allocation	80	65	70	85	

Si represents the production place(sources) and Wi represents the warehouses (destinations). We can represent the above problem with network diagram as following



In above fig, the arrows shows the possible routes for transportation, where the number next to each arrow is the shipping cost per unit for that route. A square bracket net to each location gives the number of units to be shipped out of that location (so that the allocation into each warehouse is given as a negative number)

We can also represent the above problem in term of linear programming.

With above cost structure, the linear programming problem of the firm will be

$$\text{Minimize} \quad Z = 464x_{11} + 513x_{12} + 654x_{13} + 867x_{14} + 352x_{21} + 416x_{22} + 690x_{23} + 791x_{24} + 995x_{31} + 682x_{32} + 388x_{33} + 685x_{34},$$

subject to the constraints

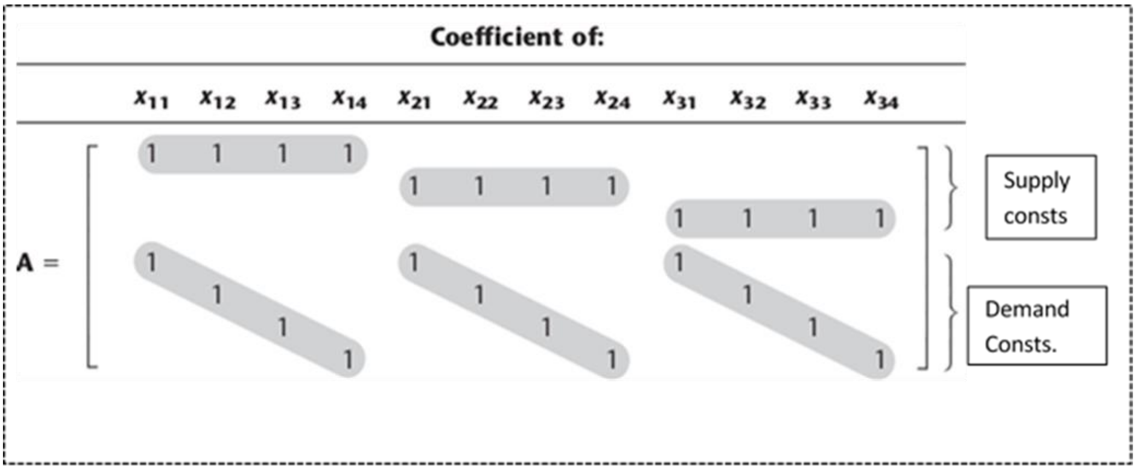
$$\begin{aligned}
&x_{11} + x_{12} + x_{13} + x_{14} = 75\\
&x_{21} + x_{22} + x_{23} + x_{24} = 125\\
&x_{31} + x_{32} + x_{33} + x_{34} = 100\\
&x_{11} + x_{21} + x_{31} = 80\\
&x_{12} + x_{22} + x_{32} = 65\\
&x_{13} + x_{23} + x_{33} = 70\\
&x_{14} + x_{24} + x_{34} = 85
\end{aligned}$$

and

$$x_{ij} \geq 0 \qquad (i = 1, 2, 3;\ j = 1, 2, 3, 4).$$

The table 2 shows the constraint coefficients. As I will explain in class, it is the special structure in the pattern of these coefficients that distinguishes this problem as a transportation problem, not its context.

Table 2



The supply constraints also called as Source constraints and the Demand constraints are also called as Warehouse constraints.

It becomes very clear from above example that coefficient table of transportation problem have very special structure. We can show the same pattern for general transportation problem with source i=1,2,…m and destination j=1,2,… n.

The unit cost of this general transportation problem will be

		cost per unit distributed				supply
		destination				
		1	2	...	n	
	1	c11	c12		c1n	S1
source	2	c21	c22		c2n	S2
	...					
	m	cm1	cm2		cmn	Sm
	demand	d1	d2		dn	

In above table c11 represents the unit cost of transportation from source 1 to destination 1. In the same way, c12 repesents the unit cost of transportation from source 1 to destination 2.

The mathematical formation for this problem will be

Min $Z = \sum_i \sum_j c_{ij} x_{ij}$

Subject to

$\sum_i x_{ij} = s_i$

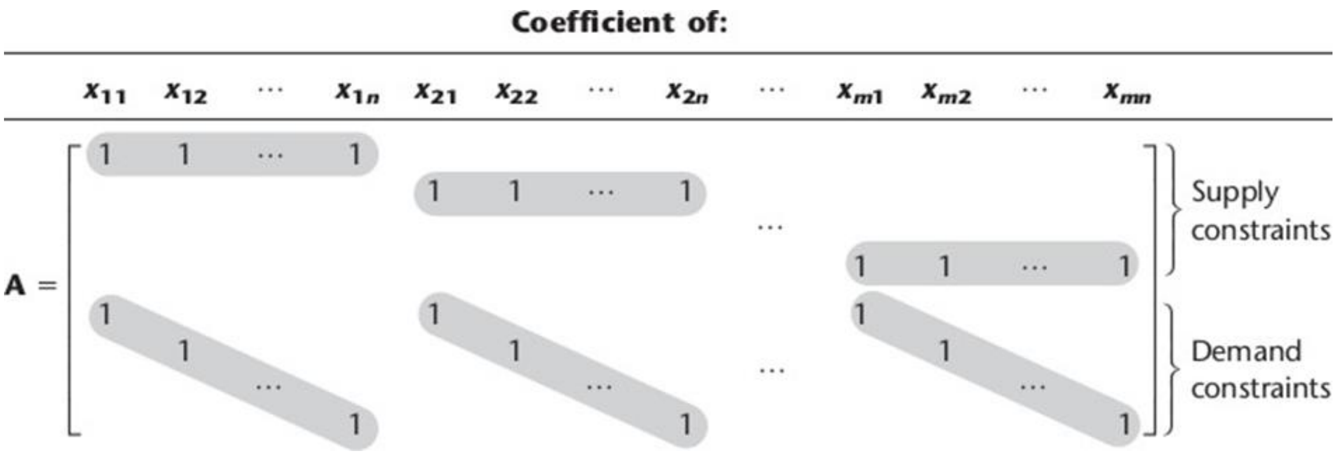
$\sum_j x_{ij} = d_j$

$x_{ij} \geq 0$ for all i and j.

In transportation model we make following two assumption.

- The requirement assumption- Each sources has a fixed supply of units, where this entire supply must be distributed to the destinations. Similarly, each destination has a fixed demand for units, where this entire demand must be received from the sources.
- The cost assumptions- the cost of distributing units from any particular source to any particular destination is directly proportional to the number of units distributed. Therefore, the cost is just the unit cost of distribution times the number of units distributed.

Note that the resulting table of constraint coefficients has the special structure shown in following table. Table 3



So any problem (whether involving transportation or not) fits the model for a transportation problem if it can be described completely in terms of a parameter table like above table and it satisfies both the requirement assumptions and the cost assumption. The objective is to minimise the total cost of distributing the units. All the parameters of the model are included in this table.

2.1.1 Basic Terminologies

1. Rim Requirement- the supply and demand requirements at various source and destinations is called Rim requirement.
2. Feasible Solution- A feasible solution to a transportation problem is a set of non-negative allocations, xij, that satisfies the rim (row and column) restrictions.
3. Basic Feasible solution- A feasible solution is called a basic feasible solution if it contains no more than m+n-1 non negative allocations where m is the number of rows and n is the number of columns of the transportation problem.
4. Degenerated basic feasible- A basic solution that contains less than m+n-1 non-negative allocations in independent positions. The allocation are said to be independent positions, if it is not possible to form a closed path (loop) means by allowing horizontal and vertical lines and if all the corner cells are occupied. (explanation in class)
5. Non-degenerated basic feasible- A basic solution that contains exactly m+n-1 non-negative allocations is called degenerated basic feasible solution.
6. Optimal Solution- A feasible solution (non necessarily basic) is said to be optimal if it minimises (maximises) the transportation cost (profit).
7. Balance and Unbalance Transportation Problem-If the total demand is equal to total supply then TP is called balance TP, otherwise it is unbalance TP
8. Occupied and Unoccupied cells- the allocated cells in the transportation cells occupied cells and empty cells are called non-occupied cells.

2.1.2 The Transportation Algorithm

The transportation algorithm follows the exact steps of the simplex method. However, instead of using the regular simplex tableau, we take advantage of the special structure of the transportation model to organise the computations in a more convenient form.

Steps- the steps of the transportation algorithm are exact parallels of the simplex algorithm,

Step 1- Determine a starting basic feasible solution, and

Step 2- use the optimality condition of the simplex method to determine the entering variable from among all the non basic variables. If the optimality condition is satisfied, stop. Otherwise go to step 3

Step 3- use the feasibility condition of the simplex method to determine the leaving variable from among all the current basic variables, and find the new basic solution. Return to step 2.

Step 1: Determination of the starting solution

A general transportation model with m sources and n destinations has m+n constraint equations, one for each source and each destination. However, because the transportation model is always balanced (sum of the supply=sum of the demad), one of these equations is redundant. Thus, the model has m+n-1 independent constraint equations, which means that the starting solution basic solution consists of m+n- 1 independent equations, which means that the starting basic solution consists of m+n-1 basic variables.

The special structure of the transportation problem allows securing a nonartificial starting basic solution using one of three methods

(1) Least cost method

(2) Vogel approximation method.

Above methods differ in the quality of the starting basic solution they produce, in the sense that a better starting solution yields a smaller objective value.

Let Us Sum Up

The Transportation Problem aims to find the most cost-effective way to distribute products from multiple suppliers to multiple consumers, ensuring supply and demand constraints are met while minimizing total transportation costs. Techniques like the Northwest Corner Rule, Least Cost Method, and Vogel Approximation Method, followed by the MODI Method, are used to find optimal solutions.

Check Your Progress

1.What is the primary objective of the Transportation Problem in Operations Research?

a) To maximize the number of shipments

b) To minimize the total transportation cost

c) To balance supply and demand exactly

d) To maximize profit margins

2.Which method is commonly used to find an initial feasible solution to the Transportation Problem?

a) Simplex Method

b) Hungarian Method

c) Northwest Corner Rule

d) Branch and Bound

3.After finding an initial feasible solution for a Transportation Problem, which method is typically used to optimize the solution?

a) Vogel Approximation Method (VAM)

b) MODI (Modified Distribution) Method

c) Least Cost Method

d) Monte Carlo Simulation

4.Which of the following methods does NOT provide an initial feasible solution to the Transportation Problem?

a) Northwest Corner Rule

b) Least Cost Method

c) Vogel Approximation Method (VAM)

d) Hungarian Method

2.2 LEAST COST METHOD

Definition: The **Least Cost Method** is another method used to obtain the initial feasible solution for the transportation problem. Here, the allocation begins with the cell which has the minimum cost. The lower cost cells are chosen over the higher-cost cell with the objective to have the least cost of transportation.

The Least Cost Method is considered to produce more optimal results than the North-west Corner because it considers the shipping cost while making the allocation, whereas the North-West corner method only considers the availability and supply requirement and allocation begin with the extreme left corner, irrespective of the shipping cost.

2.2.1 Steps

Step I write the transportation problem in a tabular form

Step II Assigns as much as possible to the cell with the smallest unit cost.

Step III the satisfied row or column is crossed out and the amount of supply and demand are adjusted accordingly. If both a row and a column are satisfied simultaneously, only one is crossed out.

Step IV Look for the uncrossed out cell with the smallest unit cost and repeat the process until exactly one row or column is left uncrossed out.

Let's understand the concept of Least Cost method through a problem given below:

Source \ To	D	E	F	Supply
A	5	8	4	50
B	6	6	3	40
C	3	9	6	60
Demand	20	95	35	150

Least Cost Method

Definition: The **Least Cost Method** is another method used to obtain the initial feasible solution for the transportation problem. Here, the allocation begins with the cell which has the minimum cost. The lower cost cells are chosen over the higher-cost cell with the objective to have the least cost of transportation.

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Let's understand the concept of Least Cost method through a problem given below:

Source \ To	D	E	F	Supply
A	5	8	4	50
B	6	6	3	40
C	3	9	6	60
Demand	20	95	35	150

In the given matrix, the supply of each source A, B, C is given Viz. 50units, 40 units, and 60 units respectively. The weekly demand for three retailers D, E, F i.e. 20 units, 95 units and 35 units is given respectively. The shipping cost is given for all the routes.

The minimum transportation cost can be obtained by following the steps given below:

Source \ To	D	E	F	Supply
A	5	8 (50)	4	50
B	6	6 (5)	3 (35)	40
C	3 (20)	9 (40)	6	60
Demand	20	95	35	150

1. The minimum cost in the matrix is Rs 3, but there is a tie in the cell BF, and CD, now the question arises in which cell we shall allocate. Generally, the cost where maximum quantity can be assigned should be chosen to obtain the better initial solution. Therefore, 35 units shall be assigned to the cell BF. With this, the demand for retailer F gets fulfilled, and only 5 units are left with the source B.
2. Again the minimum cost in the matrix is Rs 3. Therefore, 20 units shall be assigned to the cell CD. With this, the demand of retailer D gets fulfilled. Only 40 units are left with the source C.
3. The next minimum cost is Rs 4, but however, the demand for F is completed, we will move to the next minimum cost which is 5. Again, the demand of D is completed. The next minimum cost is 6, and there is a tie between three cells. But however, no units can be assigned to the cells BD and CF as the demand for both the retailers D and F are saturated. So, we shall assign 5 units to Cell BE. With this, the supply of source B gets saturated.
4. The next minimum cost is 8, assign 50 units to the cell AE. The supply of source A gets saturated.
5. The next minimum cost is Rs 9; we shall assign 40 units to the cell CE. With his both the demand and supply of all the sources and origins gets saturated.
6. The total cost can be calculated by multiplying the assigned quantity with the concerned cost of the cell. Therefore,
7. Total Cost = 50*8 + 5*6 + 35*3 +20*3 +40*9 = Rs 955.

The supply and demand should be equal and in case supply are more, the dummy source is added in the table with demand being equal to the difference between supply and demand, and the cost remains zero. Similarly, in case the demand is more than supply, then dummy destination or origin is added to the table with the supply equal to the difference in quantity demanded and supplied and the cost being zero.

2.3 VOGEL'S APPROXIMATION METHOD (VAM)

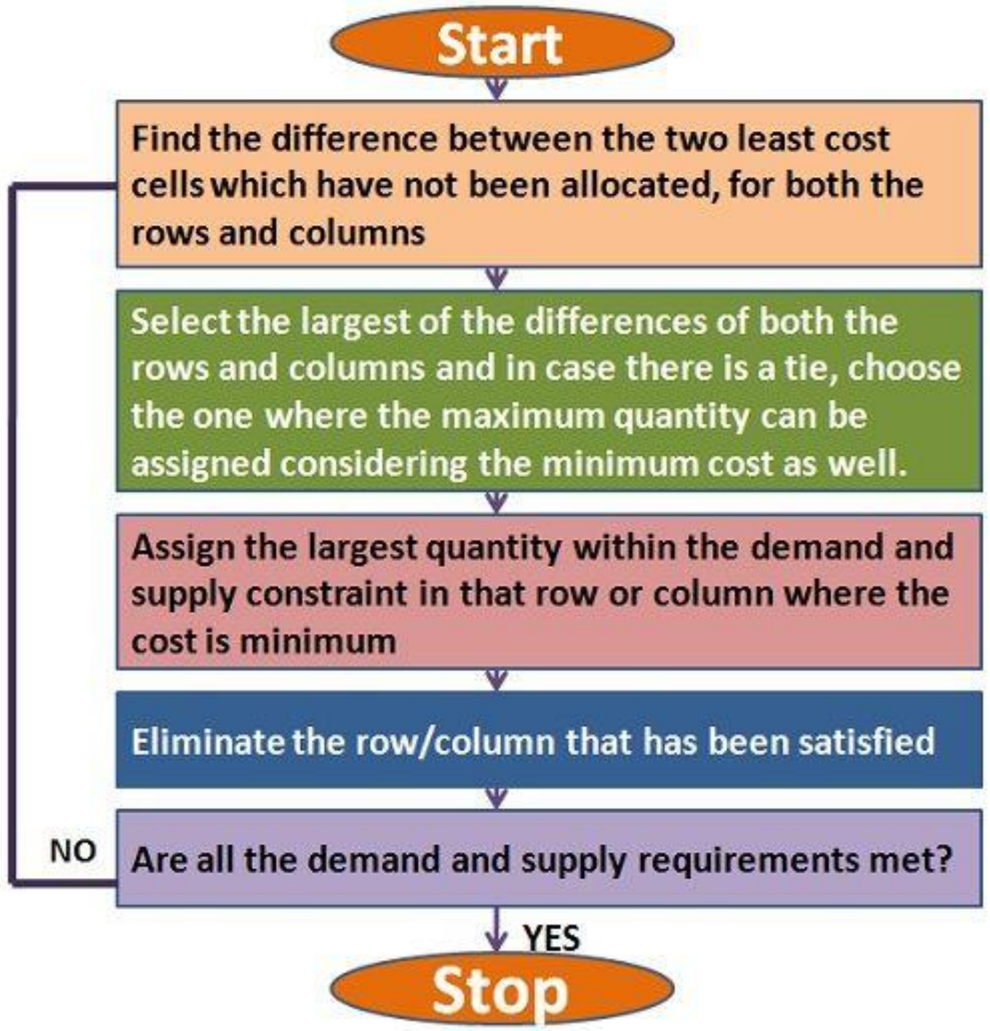
VAM is an improved version of the least cost method that generally, but not always, produces more efficient starting solutions.

Definition: The **Vogel's Approximation Method** or **VAM** is an iterative procedure calculated to find out the initial feasible solution of the transportation problem. Like Least cost Method, here also the shipping cost is taken into consideration, but in a relative sense.

Vogel's Approximation Method

Definition: The **Vogel's Approximation Method** or **VAM** is an iterative procedure calculated to find out the initial feasible solution of the transportation problem. Like Least cost Method, here also the shipping cost is taken into consideration, but in a relative sense.

The following is the flow chart showing the steps involved in solving the transportation problem using the Vogel's Approximation Method:



2.3.1 Steps

Step I Write the TP in the tabular form

Step II For each row (column), determine a penalty measure by subtracting the smallest unit cost element in the row (column) from the next smallest unit cost element in the same row (column)

Step III Identify the row or column with the largest penalty. Break ties arbitrarily. Allocate as much as possible to the variable with the least unit cost in the selected row or column. Adjust the supply and demand, and cross out the satisfied row or column. If a row and a column are satisfied simultaneously, only one of the two is crossed out, and the remaining row (column) is assigned zero supply (demand).

Step IV (a) if exactly one row or column with zero supply or demand remains uncrossed out, stop.

(b) if one row (column) with positive supply (demand) remains uncrossed out, determine the basic variable in the row (column) by the least cost method. Stop

(c)if all the uncrossed out rows and columns have (remaining) zero supply and demand,

Determine the zero basic variables by the least cost method. Stop

(d) Otherwise, go to step II

VAM is popular because it is relatively easy to implement by hand.

The penalty in VAM represents the minimum extra unit cost incurred by failing to make an allocation to the cell having the smallest unit in that row or column, this criterion does take costs into account in an effective way.

Vogel's Approximation Method can be well understood through an **illustration given below:**

1. First of all the difference between two least cost cells are calculated for each row and column, which can be seen in the iteration given for each row and column. Then the largest difference is selected, which is 4 in this case. So, allocate 20 units to cell BD, since the minimum cost is to be chosen for the allocation. Now, only 20 units are left with the source B.

From \ To	D	E	F	Supply	Iteration-I
A	6	4	1	**50**	3
B	3 (20)	8	7	**40**	(4)
C	4	4	2	**60**	2
Demand	**20**	**95**	**35**	**150**	
Iteration-I	1	0	1		

2. Column D is deleted, again the difference between the least cost cells is calculated for each row and column, as seen in the iteration below. The largest difference value comes to be 3, so allocate 35 units to cell AF and 15 units to the cell AE. With this, the Supply and demand of source A and origin F gets saturated, so delete both the row A and Column F.

From \ To	E	F	Supply	Iteration-II
A	4 (15)	1 (35)	50	(3)
B	8	7	20	1
C	4	2	60	2
Demand	95	35	150	
Iteration-II	0	1		

3. Now, single column E is left, since no difference can be found out, so allocate 60 units to the cell CE and 20 units to cell BE, as only 20 units are left with source B. Hence the demand and supply are completely met.

From \ To	E	Supply
B	8 (20)	20
C	4 (60)	60
Demand	80	150

Now the total cost can be computed, by multiplying the units assigned to each cell with the cost concerned. Therefore,

Total Cost = 20*3 + 35*1 + 15*4 + 60*4 + 20*8 = Rs 555

Note: Vogel's Approximation Method is also called as **Penalty Method** because the difference costs chosen are nothing but the penalties of not choosing the least cost routes.

Let Us Sum Up

Least cost and Vogel's approximation method are playing pivotal role in transportation calculation

Check Your Progress

1.What is the primary objective of the Least Cost Method in the Transportation Problem?

a) To find the optimal solution directly

b) To maximize the profit

c) To provide an initial feasible solution with a low transportation cost

d) To balance supply and demand exactly

2.In the Least Cost Method, how are shipments allocated?

a) By selecting the route with the highest cost first

b) By selecting the route with the least cost first

c) By balancing supply and demand simultaneously

d) By random allocation

3. What is the primary objective of Vogel's Approximation Method (VAM) in the Transportation Problem?

a) To find the optimal solution directly

b) To minimize the penalties associated with unfulfilled demand

c) To provide an initial feasible solution that is closer to the optimal solution

d) To equalize supply and demand exactly

4. How does Vogel's Approximation Method determine the penalty for each row and column?

a) By subtracting the smallest cost from the largest cost in each row and column

b) By subtracting the smallest cost from the second smallest cost in each row and column

c) By averaging the costs in each row and column

d) By summing all the costs in each row and column

5. In Vogel's Approximation Method, after determining the penalties, how are shipments allocated?

a) By selecting the route with the smallest penalty

b) By selecting the route with the highest penalty

c) By selecting the route with the least cost among the highest penalties

d) By randomly assigning routes

2.4 UNBALANCED TRANSPORTATION MODEL

A necessary and sufficient condition for the existence of feasible solution to the general transportation problem is that the total demand must equal the total supply. However, sometimes there may be more demand than the supply and vice versa in which case the problem is said to be unbalanced. If may occur in the following situation

1. SS > DD
2. DD > SS

In case I, we introduce a dummy destination in the transportation table. The cost of transportating to this destination are all set equal to zero. The requirement at this dummy destination is then assumed to be equal to

SS – DD

In case 2, we introduce a dummy source in the transportation table. The cost of transporting from this source to any destinations is all set equal to zero. The availability at this dummy source is assumed to be equal to DD – SS.

Step 2 and 3: Test of optimality and optimal solution

Once an initial solution is obtained, the next step is to check its optimality. An optimal solution is one where there is no other set of transportation route (allocations) that will further reduce the total transportation cost. Thus we have to evaluate each unoccupied cell (represents unused route) in the transportation table in terms of an opportunity of reducing total transportation cost.

There are two methods to check optimality-

1. Stepping Stone Method
2. Modified distribution (MODI)

2.4.1 Stepping Stone Method

This is a procedure for determining the potential of improving upon each of the non-basic variables in terms of the objective function. To determine this potential, each of the non-basic variables is considered one by one. For each such cell, we find what effect on the total cost would be if one unit is assigned to this cell. With this information, then, we come to know whether the solution is optimal or not. If not, we improve that solution.

Definition: The Stepping Stone Method is used to check the optimality of the initial feasible solution determined by using any of the method Viz. North-West Corner, Least Cost Method or Vogel's Approximation Method. Thus, the stepping stone method is a procedure for finding the potential of any non-basic variables (empty cells) in terms of the objective function.

Through Stepping stone method, we determine that what effect on the transportation cost would be in case one unit is assigned to the empty cell. With the help of this method, we come to know whether the solution is optimal or not.

The series of steps are involved in checking the optimality of the initial feasible solution using the stepping stone method:

1. The prerequisite condition to solve for the optimality is to ensure that the number of occupied cells is exactly equal to m+n-1, where 'm' is the number of rows, while 'n' is equal to the number of columns.
2. Firstly, the empty cell is selected and then the closed path is created which starts from the unoccupied cell and returns to the same unoccupied cell, called as a "closed loop". For creating a closed loop the following conditions should be kept in mind:

- In a closed loop, cells are selected in a sequence such that one cell is unused/unoccupied, and all other cells are used/occupied.
- A pair of Consecutive used cells lies either in the same row or the same column.
- No three consecutive occupied cells can either be in the same row or column.
- The first and last cells in the closed loop lies either in the same row or column.
- Only horizontal and vertical movement is allowed.

3. Once the loop is created, assign "+" or "–" sign alternatively on each corner cell of the loop, but begin with the "+" sign for the unoccupied cell.

4. Repeat these steps again until all the unoccupied cells get evaluated.

5. Now, if all the computed changes are positive or are equal to or greater than zero, then the optimal solution has been reached.

6. But in case, if any, value comes to be negative, then there is a scope to reduce the transportation cost further. Then, select that unoccupied cell which has the most negative change and assign as many units as possible. Subtract the unit that added to the unoccupied cell from the other cells with a negative sign in a loop, to balance the demand and supply requirements.

Example, suppose the following matrix shows the initial feasible solution and stepping stone method is adopted to check its optimality

From \ TO	D	E	F	Supply
A	6 (20)	4 (30)	1	50
B	3	8 (40)	7	40
C	4	4 (25)	2 (35)	60
Demand	20	95	35	150

Optimality: m+n-1 = 3+3-1 = 5

Total Cost: 20*6 +30*4 + 40*8 +25*4 +35*2 = 730

Empty Cells: AF, BD, BF, CD

From \ To	D	E	F	Supply
A	-6 (20)	+4 (30)	1	50
B	+3	-8 (40)	7	40
C	4	4 (25)	2 (35)	60
Demand	20	95	35	150

Cell	Closed Loop	Net Cost Change	Opportunity Cost
BD	BD-AD+AE-BE	3-6+4-8 = -7	+7
AF	AF-CF+CE-AE	1-2+4-4 = -1	+1
BF	BF-CF+CE-BE	7-2+4-8 = 1	-1
CD	CD-AD+AE-CE	4-6+4-4 = -2	+2

The most favored cell is BD, since it has the highest opportunity cost i.e. 7

To / From	D	E	F	Supply
A	-6	+4 (50)	1	50
B	+3 (20)	-8 (20)	7	40
C	4	4 (25)	2 (35)	60
Demand	20	95	35	150

Total Cost: 20*3 +50*4 + 20*8 + 25*4 +35*2 = 590

With the new matrix so formed, again the empty cells will be evaluated through a loop formation and signs will be assigned accordingly. The cell with the highest opportunity cost will be assigned the units, and this process will repeat until the best optimum solution is obtained or the opportunity cost of all the unoccupied cells comes to be negative.

We can summarise the Stepping Stone method in following steps

1. Construct a transportation table with a given unit cost of transportation along with the rim conditions

2. Determine a initial basic feasible solution (allocation) using a suitable method as discussed earlier

3. Evaluate all unoccupied cells for the effect of transferring one unit from an occupied cell to the unoccupied cell. This transfer is made by forming a closed path that retains the SS and DD condition of the problem

4. Check the sign of each of the net change in the unit transportation costs. If the net changes are plus or zero, then the an optimal solution has been arrived at, otherwise go to step 5

5. Select the unoccupied cell with most negative net change among all unoccupied cells.

6. Assign an many units as possible to unoccupied cell satisfying rim conditions. The maximum number of units to be assigned are equal to the smaller circled number among the occupied cells with the minus value in a closed path.

7. Go to step 3, and repeat the problem until all unoccupied cells are evaluated and the net change result in positive or zero.

2.4.2 Modified distribution (MODI)

The Modified Distribution Method, also known as MODI method or u-v method, which provides a minimum cost solution (optimal solution) to the transportation problem. The following are the steps involved in this method

Definition: The Modified Distribution Method or MODI is an efficient method of checking the optimality of the initial feasible solution.

Step 1: Find out the basic feasible solution of the transportation problem using any one of the three methods discussed in the previous section.

Step 2: Introduce dual variables corresponding to the row constraints and the column constraints. If there are m origins and n destinations then there will be m+n dual variables. The dual variables corresponding to the row constraints are represented by Ui, i = 1,2,…..m where as the dual variables corresponding to the column constraints are represented by Vj, j = 1,2,…..n. The values of the dual variables are calculated from the equation given below Ui + Vj = Cij if Xij > 0

Step 3: Any basic feasible solution has m + n -1 and Xij > 0. Thus, there will be m + n -1 equation to determine m + n dual variables. One of the dual variables can be chosen arbitrarily. It is also to be noted that as the primal constraints are equations, the dual variables are unrestricted in sign.

Step 4: If Xij =0, the dual variables calculated in Step 3 are compared with the C ij values of this allocation as Cij – Ui – Vj. If al Cij – Ui – Vj ≥ 0, then by the theorem of complementary slackness it can be shown that the corresponding solution of the transportation problem is optimum. If one or more Cij – Ui – Vj < 0, we select the cell with the least value of Cij – Ui – Vj and allocate as much as possible subject to the row and column constraints. The allocations of the number of adjacent cell are adjusted so that a basic variable becomes non-basic.

Step 5: A fresh set of dual variables are calculated and repeat the entire procedure from Step 1 to Step 5

The concept of MODI can be further comprehended through an illustration given below:

Initial basic feasible solution is given below:

From \ TO	D	E	F	Supply
A	6 (20)	4 (30)	1	50
B	3	8 (40)	7	40
C	4	4 (25)	2 (35)	60
Demand	20	95	35	150

Optimality: m+n-1 = 3+3-1 = 5

.

Total Cost: 20*6 +30*4 + 40*8 +25*4 +35*2 = 730

Now, calculate the values of ui and vj by using the equation:

ui+vj = Cij

Substituting the value of u1 as 0

U1+V1 = C11, 0+V1 = 6 or V1 = 6

U1 +V2 = C12, 0+V2 = 4 or V2 = 4

U2+V2 = C22, U2+4 = 8 or U2 = 4

U3+ V2 = C32, U3+4 = 4 or U3 = 0

U3+V3 = C33, 0+V3 = 2 or V3 =2

	V1	V2	V3	V4		
	D	E	F	Supply	ui	
A	6 (20)	4 (30)	1	50	0	u1
B	3	8 (40)	7	40	4	u2
C	4	4 (25)	2 (35)	60	0	u3
Demand	20	95	35	150		
vj	6	4	2			

Next step is to calculate the opportunity cost of the unoccupied cells (AF, BD, BF, CD) by using the following formula:

Cij – (ui+Vi)

AF = C13 – (U1+V3), 1- (0+2) = -1 or 1

BD = C21 – (U2+v1), 3- (4+6) = -7 or 7

BF = C23 – (U2+V3), 7- (4+2) = 1 or -1

CD = C31- (U3+V1), 4- (0+6) = -2 or 2

Choose the largest positive opportunity cost, which is 7 and draw a closed path, as shown in the matrix below. Start from the unoccupied cell and assign "+" or "–"sign alternatively. Therefore, The most favored cell is BD, assign as many units as possible.

	V1	V2	V3	V4		
	D	E	F	Supply	ui	
A	(20) -6	(30) +4	1	50	0	u1
B	+3	(40) -8	7	40	4	u2
C	4	(25) 4	(35) 2	60	0	u3
Demand	20	95	35	150		
vj	6	4	2			

The matrix below shows the maximum allocation to the cell BD, and that number of units are added to the cell with a positive sign and subtracted from the cell with a negative sign.

	V1	V2	V3	V4		
	D	E	F	Supply	ui	
A	6	(50) 4	1	50	0	u1
B	(20) 3	(20) 8	7	40	4	u2
C	4	(25) 4	(35) 2	60	0	u3
Demand	20	95	35	150		
vj	6	4	2			

Again, repeat the steps from 1 to 4 i.e. find out the opportunity costs for each unoccupied cell and assign the maximum possible units to the cell having the largest opportunity cost. This process will go on until the optimum solution is reached.

The Modified distribution method is an improvement over the stepping stone method since; it can be applied more efficiently when a large number of sources and destinations are involved, which becomes quite difficult or tedious in case of stepping stone method.

Modified distribution method reduces the number of steps involved in the evaluation of empty cells, thereby minimizes the complexity and gives a straightforward computational scheme through which the opportunity cost of each empty cell can be determined.

2.5 ASSIGNMENT PROBLEM

The AP is a special type of LPP where assignees are being assigned to perform task. For example, the assignees might be employees who need to be given work assignments. However, the assignees might not be people. They could be machines or vehicles or plants or even time slots to be assigned tasks.

To fit the definition of an assignment problem, the problem need to formulate in a way that satisfies the following assumptions

1. The number of assignees and the number of tasks are the same
2. Each assignees is to be assigned to exactly one task
3. Each task is to performed by exactly one assignee
4. There is a cost cij associated with assignee i performing task j.
5. The objective is to determine how all n assignments should be made to minimise the total cost.

Any problem satisfying all these assumptions can be solved extremely efficiently by algorithm designed specifically for assignment problem.

Given n facilities, n jobs and the effectiveness of each facility to each job, here the problem is to assign each facility to one and only one job so that the measure of effectiveness if optimized. Here the optimization means Maximized or Minimized

There are many management problems has a assignment problem structure.

For example, the head of the department may have 6 people available for assignment and 6 jobs to fill. Here the head may like to know which job should be assigned to which person so that all tasks can be accomplished in the shortest time possible.

Another example a container company may have an empty container in each of the location 1, 2,3,4,5 and requires an empty container in each of the locations 6, 7, 8,9,10. It would like to ascertain the assignments of containers to various locations so as to minimize the total distance.

The third example here is, a marketing set up by making an estimate of sales performance for different salesmen as well as for different cities one could assign a particular salesman to a particular city with a view to maximize the overall sales.

Note that with n facilities and n jobs there are n! possible assignments.

The simplest wayof finding an optimum assignment is to write all the n! possible arrangements, evaluate their total cost and select the assignment with minimum cost. But this method leads to a Calculation problem of formidable size even when the value of n is moderate.

2.5.1 Assignment Problem Structure and Solution

The mathematical model for the assignment problem uses the following decision variables X_{ij} = 1 if assignee(worker) i perform task j

=0 if not.

Thus each x_{ij} is a binary variable (it has value 0 or 1). Lets Z denots the total cost, the assignment problem model is

Min $Z= \sum_i \sum_j c_{ij} x_{ij}$

Subject to

$\sum_i x_{ij}=1$ for i= 1. n

$\sum_j x_{ij}=1$ for j=1 n

2.5.2 Transportation Problem vs. Assignment Problem

Transportation Problem	Assignment Problem
It is used to optimize the transportation cost.	t is about assigning a finite source to a fin destination (one source is alloted to one destination).
A number of Sources and demands may or may not be equal.	The number of sources and the number c destinations must be equal.
If demand and supply are not equal, then the transportation problem is known as the Unbalance Transportation Problem.	If the number of rows and the number of columns are not equal, then the assignment problem is known as the Unbalanced Assignment Problem.
It requires two steps to solve: Find the Initial Solution using North West, Least Cost or Vogel Approximation Find Optimal Solution using the MODI method.	It requires only one step to solve. he Hungarian Method is sufficient to find t optimal solutions.

2.6 HUNGARIAN METHOD

The Hungarian Method is discussed in the form of a series of computational steps as follows, when the objective function is that of minimization type.

Step 1: From the given problem, find out the cost table. Note that if the number of origins is not equal to the number of destinations then a dummy origin or destination must be added.

Step 2: In each row of the table find out the smallest cost element, subtract this smallest cost element from each element in that row. So, that there will be at least one zero in each row of the new table. This new table is known as First Reduced Cost Table.

Step 3: In each column of the table find out the smallest cost element, subtract this smallest cost element from each element in that column. As a result of this, each row and column has at least one zero element. This new table is known as Second Reduced Cost Table.

Step 4: Now determine an assignment as follows:

i. For each row or column with a single zero element cell that has not be assigned or eliminated, box that zero element as an assigned cell.

ii. For every zero that becomes assigned, cross out all other zeros in the same row and for column.

iii. If for a row and for a column there are two or more zero and one can't be chosen by inspection, choose the assigned zero cell arbitrarily.

iv. The above procedures may be repeated until every zero element cell is either assigned(boxed) or crossed out.

Step 5: An optimum assignment is found, if the number of assigned cells is equal to the number of rows (and columns). In case we had chosen a zero cell arbitrarily, there may be an alternate optimum. If no optimum solution is found i.e. some rows or columns without an assignment then go to Step 6.

Step 6:Draw a set of lines equal to the number of assignments which has been made in Step 4, covering all the zeros in the following manner

- Mark check (√) to those rows where no assignment has been made
  - Examine the checked (√) rows. If any zero element cell occurs in those rows, check (√) the respective columns that contains those zeros
  - Examine the checked (√) columns. If any assigned zero element occurs in those columns, check (√) the respective rows that contain those assigned zeros.
  - The process may be repeated until now more rows or column can be checked.
  - Draw lines through all unchecked rows and through all checked columns.

Step 7 : Examine those elements that are not covered by a line. Choose the smallest of these elements and subtract this smallest from all the elements that do not have a line through them, Add this smallest element to every element that lies at the intersection of two lines. Then the resulting matrix is a new revised cost table

2.7 UNBALANCED ASSIGNMENT PROBLEM

In the previous section we assumed that the number of persons to be assigned and the number of jobs were same. Such kind of assignment problem is called as balanced assignment problem.

Suppose if the number of person is different from the number of jobs then the assignment problem is called as unbalanced.

If the number of jobs is less than the number of persons, some of them can't be assigned any job. So that we have to introduce on or more dummy jobs of zero duration to make the unbalanced assignment problem into balanced assignment problem.

This balanced assignment problem can be solved by usingthe Hungarian Method as discussed in the previous section. The persons to whom the dummy jobs are assigned are left out of assignment. Similarly, if the number of persons is less than number of jobs then we have introduce one or more

dummy persons with zero duration to modify the unbalanced into balanced and then the problem is solved using the Hungarian Method. Here the jobs assigned to the dummy persons are left out.

2.8 INFEASIBLE ASSIGNMENT PROBLEM

Sometimes it is possible a particular person is incapable of performing certain job or a specific job can't be performed on a particular machine. In this case the solution of the problem takes into account of these restrictions so that the infeasible assignment can be avoided.

The infeasible assignment can be avoided by assigning a very high cost to the cells where assignments are restricted or prohibited.

Check Your Progress

1.What is a characteristic of a balanced assignment problem?

a) The number of agents is equal to the number of tasks

b) The number of agents is greater than the number of tasks

c) The number of tasks is greater than the number of agents

d) There is no restriction on the number of agents and tasks

2.Which method is commonly used to solve a balanced assignment problem optimally?

a) Simplex Method

b) Hungarian Method

c) Least Cost Method

d) Branch and Bound

3.In an unbalanced assignment problem, what does it mean when there are more agents than tasks?

a) Some agents will be idle

b) Some tasks will be left unassigned

c) The problem cannot be solved

d) There will be extra costs incurred

4.How does the Hungarian Method handle an unbalanced assignment problem where there are more agents than tasks?

a) By adding dummy tasks with zero costs

b) By adding dummy agents with zero costs

c) By discarding extra agents randomly

d) By increasing the total assignment cost

5.Which of the following is a characteristic of an unbalanced assignment problem?

a) There are equal numbers of agents and tasks

b) There are fewer agents than tasks

c) There are no constraints on the number of agents and tasks

d) There are only integer values for agents and tasks

Unit Summary

Transportation Problem involves determining the most cost-effective way to distribute goods from multiple suppliers to multiple consumers while satisfying supply and demand constraints. Techniques such as the Northwest Corner Rule, Least Cost Method, and Vogel's Approximation Method are used to find initial feasible solutions, followed by optimization methods like the MODI (Modified Distribution) Method. The goal is to minimize total transportation costs and ensure efficient distribution networks. on contrast, the Assignment Problem involves assigning tasks to resources (agents) in such a way that each task is handled by exactly one resource and vice versa, typically with the objective of minimizing costs or maximizing effectiveness. The Hungarian Method is a widely used technique for solving this problem optimally, providing an efficient allocation of tasks to agents.

Glossary

Least Cost Method:
Another heuristic method for solving the Transportation Problem, where allocations are made starting with the cell that has the lowest shipping cost. This method aims to minimize the total transportation cost by sequentially assigning shipments based on the least cost per unit.

Vogel's Approximation Method (VAM):
A method used to find an initial feasible solution for the Transportation Problem. VAM compares the penalties associated with the least and second least costs in each row and column of the cost matrix. It assigns shipments to cells with the largest penalties, aiming to provide a solution closer to the optimal solution.

Modified Distribution (MODI) Method:
An optimization technique used after finding an initial feasible solution to the Transportation Problem. MODI identifies and evaluates opportunities to improve the

current solution by calculating the marginal costs associated with each route and adjusting allocations accordingly to achieve the optimal transportation cost.

Assignment Problem:

The Assignment Problem involves assigning a set of tasks to a set of agents (resources) in such a way that each task is handled by exactly one agent and each agent handles exactly one task. The objective is typically to minimize costs or maximize efficiency, and it is commonly solved using algorithms such as the Hungarian Method.

Hungarian Method:

A combinatorial optimization algorithm used to solve the Assignment Problem optimally. It efficiently finds the optimal assignment by reducing the problem to a series of augmenting paths in a bipartite graph representation of the problem, ensuring that the assignment is both feasible and optimal.

Balanced Assignment Problem:

An assignment problem where the number of agents (resources) is equal to the number of tasks, ensuring that all tasks can be assigned without leaving any resources idle.

Unbalanced Assignment Problem:

An assignment problem where the number of agents does not equal the number of tasks, resulting in some tasks potentially being left unassigned or requiring the introduction of dummy resources or tasks to balance the problem.

Dummy Resource/Task:

In the context of unbalanced assignment problems, a dummy resource or task is introduced with zero cost to balance the problem and enable the application of algorithms designed for balanced assignments.

Self-Assessment Questions

1. A company has to transport goods from three warehouses (A, B, C) to four distribution centers (1, 2, 3, 4). The supply from warehouses and demand from distribution centers are as follows:

- Warehouse A: 100 units
- Warehouse B: 150 units
- Warehouse C: 200 units
- Distribution Center 1: 50 units
- Distribution Center 2: 120 units
- Distribution Center 3: 80 units
- Distribution Center 4: 200 units – solve using north west corner rule.

2. A farmer has to distribute his produce to three different markets (X, Y, Z). The supply and demand at each location are as follows:

- Farm: 200 units
- Market X: 100 units
- Market Y: 150 units
- Market Z: 120 units– solve using north west corner rule.

3. A manufacturing plant produces three types of products (P, Q, R) which need to be transported to

four different locations (W, X, Y, Z). The supply and demand are as follows:

- Product P: 80 units
- Product Q: 120 units
- Product R: 150 units
- Location W: 50 units
- Location X: 100 units
- Location Y: 80 units

- Location Z: 200 units– solve using north west corner rule.

4. A bookstore has to deliver books from two warehouses (Warehouse 1, Warehouse 2) to three different bookstores (A, B, C). The supply and demand are as follows:

- Warehouse 1: 80 units
- Warehouse 2: 120 units
- Bookstore A: 50 units
- Bookstore B: 100 units
- Bookstore C: 150 units -solve using north west corner rule.

5. An ice cream manufacturer has to distribute ice cream to five different stores (S1, S2, S3, S4, S5). The supply and demand are as follows:

- Manufacturer: 300 units
- Store S1: 100 units
- Store S2: 80 units
- Store S3: 120 units
- Store S4: 150 units
- Store S5: 200 units-solve using north west corner rule.

Exercise

6. Find Solution using Least Cost method

D1	D2	D3	D4	Supply	
S1	19	30	50	10	7
S2	70	30	40	60	9
S3	40	8	70	20	18
Demand		5	8	7	14

7. Obtain an initial basic feasible solution to the following transportation problem using least cost method.

	D_1	D_2	D_3	D_4	Supply
O_1	1	2	3	4	6
O_2	4	3	2	5	8
O_3	5	2	2	1	10
Demand	4	6	8	6	

Here Oi and Dj denote ith origin and jth destination respectively.

8. Determine how much quantity should be stepped from factory to various destinations for the following transportation problem using the least cost method

		Destination				
		C	*H*	*K*	*P*	Capacity
Factory	*T*	6	8	8	5	30
	B	5	11	9	7	40
	M	8	9	7	13	50
	Demand	35	28	32	25	

Cost are expressed in terms of rupees per unit shipped.

9. solve using Vogel's Approximation Method:

Supply D1 D2 D3

S1 20 2 3 1

S2 30 4 2 5

S3 25 3 4 2

Demand 10 20 25

10. solve using Vogel's Approximation Method:

	Supply	D1	D2	D3	D4
S1	15	8	6	10	9
S2	25	9	12	13	7
S3	10	14	10	11	8
Demand		5	15	15	15

11. solve using Vogel's Approximation Method:

	Supply	D1	D2	D3
S1	10	7	5	6
S2	35	8	6	4
S3	40	6	7	5
Demand		20	30	35

12. solve using Vogel's Approximation Method:

Supply	D1	D2	D3	D4	
S1	20	9	7	8	6
S2	30	6	5	9	7
S3	40	4	6	7	5
Demand		15	25	20	30

13. Solve using stepping stone method

Supply and Demand Matrix:

- Supply: Warehouse A (20 units), Warehouse B (30 units)
- Demand: Store 1 (15 units), Store 2 (25 units), Store 3 (10 units)

Cost Matrix

	Store 1	**Store 2**	**Store 3**
A	$8	$6	$10
B	$9	$12	$13

13. Solve using stepping stone method

Supply and Demand Matrix:

- Supply: Factory X (50 units), Factory Y (40 units)
- Demand: Retailer 1 (30 units), Retailer 2 (25 units), Retailer 3 (35 units)

Cost Matrix:

	Retailer 1	Retailer 2	Retailer 3
X	$4	$8	$7
Y	$5	$6	$9

14. Solve using stepping stone method

14. Supply and Demand Matrix:

- Supply: Plant 1 (40 units), Plant 2 (60 units)
- Demand: Market A (30 units), Market B (50 units), Market C (20 units)

Cost Matrix:

	Market A	Market B	Market C
1	$11	$9	$12
2	$13	$7	$10

15. Solve using stepping stone method

Supply and Demand Matrix:

Supply: Depot 1 (70 units), Depot 2 (50 units)

Demand: Location X (40 units), Location Y (60 units), Location Z (20 units)

Cost Matrix:

	Location X	**Location Y**	**Location Z**
1	$5	$8	$7
2	$6	$9	$4

16. A work shop contains four persons available for work on the four jobs. Only one person can work on any one job. The following table shows the cost of assigning each person to each job. The objective is to assign person to jobs such that the total assignment cost is a minimum.

		Jobs			
		1	**2**	**3**	**4**
Persons	**A**	20	25	22	28
	B	15	18	23	17
	C	19	17	21	24
	D	25	23	24	24

17. Solve the following unbalanced assignment problem of minimizing the total time for performing all the jobs

		Jobs 1	2	3	4	5
	A	5	2	4	2	5
	B	2	4	7	6	6
Workers	C	6	7	5	8	7
	D	5	2	3	3	4
	E	8	3	7	8	6
	F	3	6	3	5	7

18. Solve the following unbalanced assignment problem of minimizing the total time for performing all the jobs

		Jobs 1	2	3	4	5
	A	5	2	4	2	5
	B	2	4	7	6	6
Workers	C	6	7	5	8	7
	D	5	2	3	3	4
	E	8	3	7	8	6
	F	3	6	3	5	7

19. A computer centre has five jobs to be done and has five computer machines to perform them. The cost of processing of each job on any machine is shown in the table below

		Jobs				
		1	2	3	4	5
Computer Machines	1	70	30	X	60	30
	2	X	70	50	30	30
	3	60	X	50	70	60
	4	60	70	20	40	X
	5	30	30	40	X	70

Check Your Progress -Answers

1.1

1. b) To minimize the total transportation cost
2. c) Northwest Corner Rule
3. b) MODI (Modified Distribution) Method
4. d) Hungarian Method

1.2

1. c) To provide an initial feasible solution with a low transportation cost
2. b) By selecting the route with the least cost first
3. c) To provide an initial feasible solution that is closer to the optimal solution
4. b) By subtracting the smallest cost from the second smallest cost in each row and column
5. c) By selecting the route with the least cost among the highest penalties

1.3

1. The number of agents is equal to the number of tasks
2. Hungarian Method
3. Some tasks will be left unassigned
4. By adding dummy tasks with zero costs
5. There are fewer agents than tasks

UNIT III

UNIT INTRODUCTION

Game Theory is a branch of mathematics and economics that studies strategic interactions between rational decision-makers, known as players, who aim to maximize their outcomes in various competitive or cooperative situations. It provides a formal framework for analyzing how individuals, firms, or nations make decisions in situations where their choices depend on the actions of others. The fundamental concept in game theory is the "game," which consists of players, their strategies, and the payoffs or outcomes associated with each combination of strategies chosen by the players. Games can be classified based on their outcomes, such as cooperative games where players collaborate to achieve mutual benefits, or non-cooperative games where players compete independently. Key components of game theory include equilibrium concepts such as Nash equilibrium, where no player can improve their payoff by unilaterally changing their strategy, assuming others' strategies remain unchanged. The Prisoner's Dilemma, a classic example in game theory, illustrates the tension between individual rationality and collective cooperation. Game theory has applications in various fields including economics, political science, biology, and computer science. It provides insights into strategic decision-making, negotiation tactics, auction designs, and conflict resolution strategies. By studying game theory, analysts and decision-makers gain valuable tools to model, predict, and understand the behavior of complex systems where strategic interactions play a crucial role in shaping outcomes.

3.GAME THEORY-INTRODUCTION

In “Game Theory”, the word game is not used in the way it is commonly used in different types of sports such as hockey, cricket, football, etc. Also, it does not refer to computer games. In the usual sports or games, the main objective of the opponents is to win the game. But in the games discussed under game theory between two opposing parties with conflicting interests, winning means selecting an optimal strategy, e.g., selecting an optimal course of action as we have discussed in Units 9 and 10. Game theory deals with decision making processes of players in conflicting and competitive situations where the strategy (or action or move) of a player depends upon the move of the opponent.

four types of environments under which a decision maker may have to make decisions, namely:

- Decision making under certainty
- Decision making under uncertainty
- Decision making under risk; and
- Decision making under conflict.

On the basis of whether a saddle point exists in the game or not, games can be further classified as:

- Games with saddle point, and
- Games without saddle point.

In this section, we introduce the key terms and terminology used commonly in game theory. Then we explain what is meant by a game in game theory. But before doing so, let us consider the following situation:

Suppose two children X and Y agree that:

- Each one of them will simultaneously place a coin on the table.
- Each one of them will show the outcome (head or tail).
- If the faces of both coins match (i.e., either both coins show head or both show tail), child X wins and gets Rs 1 from child Y.
- If the coins do not match, child Y wins and gets Rs 1 from child X.

We can present this information in the form of a matrix as shown below. The first numeral in the four cells having entries (1, □1), (□1, 1), (□1, 1), (1, □1) ,

respectively, represents the amount won by child X and the second numeral represents the amount won by child Y. When child X wins Rs 1 from child Y, the winning amount for X is 1 and the winning amount for Y is □ 1 (i.e., loss of 1).

		Child Y (Player II)	
		Head	Tail
Child X (Player I)	Head	1, −1	−1, 1
	Tail	−1, 1	1, −1

In game theory, the situation discussed above is known as Coin Matching Game and its solution is provided in E2 in Sec. 12.8 of Unit 12.

Let us consider another situation.

Two persons, say X and Y, are arrested by the police with enough evidence for a minor crime. The police suspect that they are responsible for a murder, but do not have enough evidence. Both persons are put in separate cells so that they have no way of communicating with each other. The police starts interrogating them in separate rooms (as shown in Fig.11.1). Each person either confesses or does not confess. Also, each one knows the consequences of confession, which are given below:

- If both of them confess, both go to jail for 5 years.
- If one of them confesses and the other does not, then the one who confessed turns government's witness while the other who did not confess goes to jail for 20 years.
- If both do not confess, both go to jail for one year.

Assume that each one of them has to protect his self-interest, which means that each person tries to act in such a way that he would have to go to jail for a shorter period of time, regardless of the way the other acts. Also assume that they have no way of communicating with each other. What should X and Y do?

We can present this information in matrix form as given below. The first numeral in the four cells having entries (−5, − 5),(0, − 20),(−20, 0),(−1, −1) represents the time to be spent in prison by person X and the second numeral represents the time to be spent in prison by person Y. A negative sign is attached to the numerals because the time spent in prison is similar to a loss.

		Person Y (Player II)	
		Confesses	Does not Confess
Person X (Player I)	Confesses	–5, –5	0, –20
	Does not Confess	–20, 0	–1, –1

In game theory, the situation discussed above is known as the Prisoner's Dilemma. Its solution is provided in following Example

Persons X and Y facing questions of policemen in separate rooms

We now define some key terms used in game theory.

Player: A participant or competitor who makes decisions is known as a player. A player may be an individual or a group of individuals. For example, in coin matching game, child X and child Y are players, and in the Prisoner's Dilemma game, person X and person Y are players.

Action: The options available to the players are known as actions or courses of action or moves.

For example, in coin matching game, the actions are “head” and “tail”; and in the prisoner’s dilemma game, “confesses” and “does not confess” are actions.

Play: A play is said to occur when each player selects a course of action.

Strategy: A predetermined rule by which a player decides his/her course(s) of action among the actions available to him/her is known as strategy for the player. A strategy may be of two types:

Pure Strategy: A strategy is said to be a pure strategy if the player selects a particular course of action each time, i.e., if a player selects, say, the ith course of action (A) each time from among n courses of actions,

A1 , A2 ,..., An , available to him or her. This means that he/she assigns probability 1 to the ith course of action and zero probability to each of the other courses of action A1, A2 ,..., Ai□1, Ai□1,..., An . So the pure strategy

is denoted by (0, 0,..., 0, 1, 0,..., 0) having values 0 at (n □1) places

except the ith position having value 1. For example, in the prisoner’s dilemma game, person X will use pure strategy (1, 0) as you will see in the solution of this game.

3.1 Characteristics of a game

A competitive situation is a competitive game if the following properties hold good 1. The number of competitors is finite, say N. 2. A finite set of possible courses of action is available to each of the N competitors. 3. A play of the game results when each competitor selects a course of action from the set of courses available to him. In game theory we make an important assumption that all the players select their courses of action simultaneously. As a result no competitor will be in a position to know the choices of his competitors. 4. The outcome of a play consists of the particular courses of action chosen by the individual players. Each outcome leads to a set of payments, one to each player, which may be either positive, or negative, or zero.

3.1.1 Mixed Strategy

A strategy for a player is said to be a **mixed strategy** if the player selects a combination of more than one courses of action by assigning a fixed probability to each course of action. That is, if there are n courses of action Ai , (1 □ i □ n) available to the player, he/she assigns

probabilities p1, p2 ,..., pn to the courses of action A1, A2 ,..., An , respectively, such that p1 □ p2 □.............pn □ 1 and pi □ 0 for all i, 1 □i □n.

Note 1: Pure strategy is a particular case of mixed strategy because in the case of pure strategy

p_i □ 1 for some i, and p □0 for all j□ i

3.1.2 Payoff Values and Payoff Matrix

You have learnt about payoff values and payoff matrices in Unit 9. Here the payoff value means money or anything elsethat motivates players and when these payoff values are represented in matrix form, the resulting matrix is known as payoff matrix. For example, in the coin matching game, payoff values are the money that child X gets from child Y or vice versa, while in prisoner's dilemma game, the time (in years) to be spent in prison represents payoff values

Optimal Strategy: The strategy for a player, which optimises his/her payoff irrespective of the strategy of his /her competitor is known as an optimal strategy.

For example, in the coin matching game, the optimal strategy for child X is

(1/2, 1/2), which is explained in the solution of E2 in Sec. 12.8 of Unit 12. Now, we are in a position to define what we mean by a game in game theory.

Definition of Game: A competitive situation is called a game if

(i) The number of competitors, called players, is finite.

(ii) The number of possible courses of action for each player is finite. However, the courses of action need not be the same for each player.

(iii) Each player selects a course of action simultaneously from among the courses of action available to him/her without directly communicating with the other player.

(iv) Every combination of courses of action results in an outcome known as payoff value, which motivates the players. Payoff value may represent loss or gain or any other thing of interest. The payoff values may be positive, negative or zero.

n-Person Game: If the number of players involved in the game is n(□ 2) , it is known as n-person game.

Two-Person Game: If the number of players involved in the game is 2, it is known as two-person game

3.2 ZERO-SUM GAME

If the algebraic sum of the payoff values of all players after each play is zero, the game is known as a zero-sum game. Mathematically, for zero-sum game,

if a , $(1 \le i \le n)$ represents the payoff value of the i^{th} player, then $\sum^{n} a = 0$

Non-Zero-Sum Game: A game is said to be non-zero-sum game if there exists at least one play such that the algebraic sum of all the payoff values is not equal to zero. For example, the prisoner's dilemma game is a non-zero-sum game.

Two-Person Zero-Sum Game: If in a game, each payoff value of one player is negative of the payoff value of the other player, it is known as two-person zero-sum game. For example, coin matching game is a two-person zero-sum game.

In two-person zero-sum game, if we call the two players as player I and player II, we see that:

each payoff value of player II is **equal in magnitude** to the payoff value of player I but **opposite in sign**.

Hence, if the payoff value of player I is known, the payoff value of player II is automatically known. So, for a two-person zero-sum game, generally, we write the payoff values of only player I instead of writing payoff values of both players. For example, payoff matrix in the case of coin matching game can simply be written as follows

		Child Y (Player II)	
		Head	Tail
Child X (Player I)	Head	1	-1
	Tail	-1	1

Let Us Sum Up

Game theory is a mathematical framework for analyzing situations in which players (individuals, firms, countries, etc.) make decisions that are interdependent. It helps predict the outcomes of strategic interactions where the success of a player's strategy depends on the strategies chosen by others.

1.Check your Progress

1.Which of the following best describes a Nash Equilibrium?

A. A situation where one player maximizes their payoff regardless of the other player's strategy.

B. A scenario where no player can improve their payoff by unilaterally changing their strategy.

C. A game where the sum of payoffs is zero for all players.

D. A cooperative game where players collude to achieve the best possible outcome.

2.In a zero-sum game, which of the following statements is true?

A. Both players can increase their payoffs simultaneously.

B. One player's gain is exactly equal to the other player's loss.

C. Cooperation between players can lead to higher total payoffs.

D. The game always reaches a Nash Equilibrium.

3.Which of the following is an example of a sequential game?

A. The Prisoner's Dilemma

B. Rock-Paper-Scissors

C. Chess

D. Matching Pennies

4. In game theory, what does a "dominant strategy" refer to?

A. A strategy that is the best response to the strategies chosen by others.

B. A strategy that always results in the highest payoff for a player, regardless of the other players' strategies.

C. A strategy that leads to a zero-sum outcome.

D. A strategy that is only optimal in cooperative games.

5.Which of the following concepts is NOT typically associated with game theory?

A. Payoffs

B. Strategies

C. Demand and Supply

D. Players

3.3 THE MAXIMIN-MINIMAX PRINCIPLE

Under this principle, first of all, a player lists the worst possible payoff values of all strategies available to him/her. Then he/she selects the strategy corresponding to the optimum payoff value from among the worst possible payoff values.

Let us consider a two-person zero-sum game to explain this principle. The payoff matrix for the game is given below:

		Player B	
		I	**II**
Player A	**I**	1	4
	II	2	3
	III	–3	5

If a_{ij} represents the payoff value when player A chooses his/her

i^{th}, $(i = I, II, III)$ strategy and player B chooses his/her j^{th}, $(j = I, II)$ strategy, then

$a_{11} = 1$ $\Rightarrow$ Player A will get Rs 1 from player B

$a_{12} = 4$ $\Rightarrow$ Player A will get Rs 4 from player B

$a_{21} = 2 \Rightarrow$ Player A will get Rs 2 from player B

$a_{22} = -3 \Rightarrow$ Player A will get Rs 3 from player B

$a_{31} = -3 \Rightarrow$ Player A will pay Rs 3 to player B

$a_{32} = 5 \Rightarrow$ Player A will get Rs 5 from player B

The Maximin and Minimax principles are decision rules used in decision theory, game theory, and various economic and business contexts to make decisions under uncertainty and competitive situations.

3.3.1 Maximin Principle

The Maximin principle is used when a decision-maker is pessimistic or risk-averse. It involves selecting the option with the best of the worst possible outcomes. In other words, you maximize the minimum gain.

Example:

Imagine a company is deciding which of three new products to launch. The potential profits (in thousands of dollars) under three different market conditions (low, medium, high demand) are as follows:

Pr :t	**Low and**	**Medium and**	**High and**
A	20	50	80
B	30	40	60
C	10	70	90

Step-by-step:

1. Identify the worst outcome for each product:
    - Product A: Min(20, 50, 80) = 20
    - Product B: Min(30, 40, 60) = 30
    - Product C: Min(10, 70, 90) = 10

2. Select the product with the highest minimum value:
   - Max(20, 30, 10) = 30

According to the Maximin principle, the company should choose Product B, as it has the highest minimum profit (30).

3.3.2 Minimax Principle

The Minimax principle is often used in competitive scenarios (such as game theory) and focuses on minimizing the possible losses. It is about minimizing the maximum loss.

Example:

Consider two companies, A and B, deciding on their pricing strategies. The payoff matrix (profits in thousands of dollars) based on their pricing strategies (High Price, Low Price) is given below. Each cell represents the profit of Company A, with Company B's profit being the negative of that value.

	B: High	**B: Low**
A	50	-20
A	-30	10

Step-by-step:

1. Identify the maximum loss for each strategy of Company A:
   - High Price: Max(50, -20) = 50
   - Low Price: Max(-30, 10) = 10
2. Select the strategy with the smallest maximum loss:
   - Min(50, 10) = 10

According to the Minimax principle, Company A should choose the Low Price strategy to minimize its potential maximum loss, which is 10.

Steps for applying maximin-minimax principle to numerical problems

1. Construct the Payoff Matrix:
   - Create a table where rows represent the available strategies (or actions) and columns represent the possible states of nature (or outcomes).
   - Fill in the table with the corresponding payoffs for each combination of strategy and state.
2. Compute the Minimum Payoff for Each Strategy (Maximin):
   - For each row (strategy) in the payoff matrix, identify the minimum payoff across all columns (states of nature).
   - Write down these minimum payoffs.
3. Select the Maximin Strategy:
   - Compare the minimum payoffs calculated in the previous step.
   - Choose the strategy that has the highest minimum payoff.
4. (Optional) Construct the Loss Matrix:
   - If dealing with losses instead of payoffs, or if you prefer to work with losses, convert the payoff matrix to a loss matrix by negating the payoff values.
   - Alternatively, if a loss matrix is provided, use it directly.
5. Compute the Maximum Loss for Each Strategy (Minimax):
   - For each row (strategy) in the loss matrix, identify the maximum loss across all columns (states of nature).
   - Write down these maximum losses.
6. Select the Minimax Strategy:
   - Compare the maximum losses calculated in the previous step.
   - Choose the strategy that has the lowest maximum loss.

Numerical Example:

Given Payoff Matrix:

Strategy/n	Outc 1	Outc 2	Outc 3
A1	5	7	4
A2	2	6	8
A3	3	5	7

Steps for Maximin Principle:

1. Identify Minimum Payoffs:
   - For Strategy A1: min(5, 7, 4) = 4
   - For Strategy A2: min(2, 6, 8) = 2
   - For Strategy A3: min(3, 5, 7) = 3
2. Select Maximin Strategy:
   - Compare minimum payoffs: 4, 2, 3
   - Maximin Strategy: max(4, 2, 3) = 4 (Strategy A1)

Steps for Minimax Principle (using loss matrix):

1. Convert Payoff Matrix to Loss Matrix (if necessary):
   - For losses, negate the payoffs:

Strategy/n	Outc 1	Outc 2	Outc 3
A1	-5	-7	-4
A2	-2	-6	-8
A3	-3	-5	-7

2. Identify Maximum Losses:
   - For Strategy A1: max(-5, -7, -4) = -4
   - For Strategy A2: max(-2, -6, -8) = -2

- For Strategy A3: max(-3, -5, -7) = -3

3. Select Minimax Strategy:
    - Compare maximum losses: -4, -2, -3
    - Minimax Strategy: min(-4, -2, -3) = -4 (Strategy A1)

Let Us Sum Up

The minimax principle is a decision-making strategy used in game theory and decision theory. It aims to minimize the possible maximum loss in scenarios of uncertainty or conflict. The maximin principle is another decision-making strategy where a player or decision-maker aims to maximize the minimum gain (or minimize the maximum loss) under conditions of uncertainty.

Check Your Progress

1.What is the primary goal of the minimax principle?

- A) To maximize the minimum gain
- B) To minimize the maximum loss
- C) To maximize the average outcome
- D) To minimize the average loss

2.In which of the following scenarios is the maximin principle most appropriately applied?

- A) Competitive games where players have opposing goals
- B) Decision-making under certainty
- C) Decision-making under uncertainty with a focus on risk aversion
- D) Situations requiring quick, heuristic-based decisions

3.Which principle would a conservative investor most likely use when choosing between investment options?

- A) Minimax
- B) Maximin
- C) Expected Value
- D) Bayesian

4.In the context of a two-player zero-sum game, what does the minimax strategy ensure?

- A) The highest possible gain for both players
- B) The lowest possible loss for the player using the strategy
- C) A tie between the two players
- D) The maximum possible gain for the opponent

5.Which of the following best describes the difference between minimax and maximin principles?

- A) Minimax focuses on minimizing potential losses, while maximin focuses on maximizing potential gains.
- B) Minimax is used only in cooperative games, while maximin is used only in competitive games.
- C) Minimax aims to maximize average outcomes, while maximin aims to minimize average losses.
- D) Minimax is a heuristic approach, while maximin is a statistical approach.

Unit summary

Game theory is a branch of mathematics that examines strategic interactions among rational decision-makers, aiming to predict optimal choices and outcomes. Key elements include players, strategies, and payoffs, with games categorized as cooperative or non-cooperative, symmetric or asymmetric, and zero-sum or non-zero-sum. In cooperative games, players can form binding agreements, whereas, in non-cooperative games, they act independently. Zero-sum games involve one player’s gain equating to another’s loss, while non-zero-sum games allow for varying total payoffs, potentially benefiting all players.

Glossary

Zero Sum

game Because the Gain of A – Loss of B = 0. In other words, the gain of Player A is the Loss of Player B.

Pure strategy

If a player knows exactly what the other player is going to do, a deterministic situation is obtained and objective function is to minimize the gain. Therefore the pure strategy is a decision rule always to select a particular course of action.

Mixed strategy

If a player is guessing as to which activity is to be selected by the other on any particular occasion, a probabilistic situation is obtained and objective function is to maximize the expected gain. Thus, the mixed strategy is a selection among pure strategies with fixed probabilities.

Optimal strategy

The strategy that puts the player in the most preferred position irrespective of the strategy of his opponents is called an optimal strategy Any deviation from this strategy would reduce his payoff.

Saddle Point

t If the Maxi (min) of A = Mini (max) of B then it is known as the Saddle Point Saddle point is the number, which is lowest in its row and highest in its column. When minimax value is equal to maximin value , the game is said to have saddle point. It is the cell in the payoff matrix which satisfies minimax to maximin value.

Value of the Game

It is the average wining per play over a long no. of plays. It is the expected pay off when all the players adopt their optimum strategies .If the value of game is zero it is said to be a fair game , If the value of game is not zero it is said to be a unfair game . In all problems relating to game theory, first look for saddle point, then check out for rule of dominance and see if you can reduce the matrix.

Self Assessment Questions

1. You are given a game having the following payoff matrix:

		Player B		
		B_1	B_2	B_3
Player A	A_1	5	7	4
	A_2	4	2	0
	A_3	6	1	3

Obtain the

(i) Optimal strategy for player A,

(ii) Optimal strategy for player B,

(iii) Value of the game, and

(iv) Saddle point.

2. Find the maximin and minimax values for the game having the payoff matrix given below. Does the game have a saddle point? If the game has a saddle point, solve it.

Player B

B1 B2 B3

Player A A1 2 4 5

A2 6 3 7

A3 4 1 –1

3. Determine the range of values of λ and μ that will make the position (2, 2) a saddle point for the game having the payoff matrix given below:

	Player B

		B_1	B_2	B_3
Player	A_1	1	3	5
	A_2	8	4	λ
	A_3	2	μ	9

4. What are the optimal strategies for person X and person Y in the Prisoner's Dilemma game?

Exercise

5. Payoff Matrix:

Strategy	Player B1	Player B2
Player A1	3	7
Player A2	5	2

1. Determine the optimal strategy for Player A using the Maximin Principle.
2. Determine the optimal strategy for Player B using the Minimax Principle.

6. Payoff Matrix:

Strategy	Player B1	Player B2	Player B3
Player A1	4	1	6
Player A2	7	3	5

Strategy	Player B1	Player B2	Player B3
Player A3	2	8	4

1. Determine the optimal strategy for Player A using the Maximin Principle.
2. Determine the optimal strategy for Player B using the Minimax Principle.

7. Payoff Matrix:

Strategy	Player B1	Player B2	Player B3
Player A1	2	6	3
Player A2	5	2	7
Player A3	4	5	1
Player A4	3	4	8

1. Determine the optimal strategy for Player A using the Maximin Principle.
2. Determine the optimal strategy for Player B using the Minimax Principle.

8. Payoff Matrix:

Strategy	Player B1	Player B2
Player A1	1	5
Player A2	4	2

Strategy	Player B1	Player B2
Player A3	3	6

1. Determine the optimal strategy for Player A using the Maximin Principle.
2. Determine the optimal strategy for Player B using the Minimax Principle.

9. Payoff Matrix:

Strategy	Player B1	Player B2	Player B3
Player A1	8	2	4
Player A2	6	3	5
Player A3	7	1	6

1. Determine the optimal strategy for Player A using the Maximin Principle.
2. Determine the optimal strategy for Player B using the Minimax Principle.

10. Payoff Matrix:

Player A / Player B	B1	B2	B3
A1	2	3	-1
A2	1	-4	0
A3	-2	5	2

Steps:

1. Identify minimum payoffs for Player A (Maximin):
   - A1: min(2, 3, -1) = -1
   - A2: min(1, -4, 0) = -4
   - A3: min(-2, 5, 2) = -2
2. Maximin Strategy for Player A: max(-1, -4, -2) = -1 (A1)
3. Identify maximum payoffs for Player B (Minimax):
   - B1: max(2, 1, -2) = 2
   - B2: max(3, -4, 5) = 5
   - B3: max(-1, 0, 2) = 2
4. Minimax Strategy for Player B: min(2, 5, 2) = 2 (B1/B3)

11. Payoff Matrix:

Player A / Player B	B1	B2	B3
A1	4	-3	2
A2	0	1	-5
A3	-2	2	1

Steps:

1. Identify minimum payoffs for Player A (Maximin):
   - A1: min(4, -3, 2) = -3
   - A2: min(0, 1, -5) = -5
   - A3: min(-2, 2, 1) = -2
2. Maximin Strategy for Player A: max(-3, -5, -2) = -2 (A3)

3. Identify maximum payoffs for Player B (Minimax):
   - B1: max(4, 0, -2) = 4
   - B2: max(-3, 1, 2) = 2
   - B3: max(2, -5, 1) = 2
4. Minimax Strategy for Player B: min(4, 2, 2) = 2 (B2/B3)

14. Payoff Matrix:

Player A / Player B	B1	B2	B3
A1	3	-2	1
A2	5	4	-3
A3	-1	0	2

Steps:

15. Identify minimum payoffs for Player A (Maximin):
   - A1: min(3, -2, 1) = -2
   - A2: min(5, 4, -3) = -3
   - A3: min(-1, 0, 2) = -1

- Maximin Strategy for Player A: max(-2, -3, -1) = -1 (A3)
- Identify maximum payoffs for Player B (Minimax):
  - B1: max(3, 5, -1) = 5
  - B2: max(-2, 4, 0) = 4
  - B3: max(1, -3, 2) = 2
- Minimax Strategy for Player B: min(5, 4, 2) = 2 (B3)

16. Payoff Matrix:

Player A / Player B	B1	B2	B3
A1	6	2	-1
A2	3	4	0
A3	1	-2	5

Steps:

1. Identify minimum payoffs for Player A (Maximin):
   - A1: min(6, 2, -1) = -1
   - A2: min(3, 4, 0) = 0
   - A3: min(1, -2, 5) = -2
2. Maximin Strategy for Player A: max(-1, 0, -2) = 0 (A2)
3. Identify maximum payoffs for Player B (Minimax):
   - B1: max(6, 3, 1) = 6
   - B2: max(2, 4, -2) = 4
   - B3: max(-1, 0, 5) = 5
4. Minimax Strategy for Player B: min(6, 4, 5) = 4 (B2)

17. Payoff Matrix:

Player A / Player B	B1	B2	B3
A1	7	3	-2
A2	2	5	1

Player A / Player B	B1	B2	B3
A3	-3	4	2

Steps:

1. Identify minimum payoffs for Player A (Maximin):
    - A1: min(7, 3, -2) = -2
    - A2: min(2, 5, 1) = 1
    - A3: min(-3, 4, 2) = -3
2. Maximin Strategy for Player A: max(-2, 1, -3) = 1 (A2)
3. Identify maximum payoffs for Player B (Minimax):
    - B1: max(7, 2, -3) = 7
    - B2: max(3, 5, 4) = 5
    - B3: max(-2, 1, 2) = 2
4. Minimax Strategy for Player B: min(7, 5, 2) = 2 (B3)

In each of these problems, you follow the Maximin-Minimax Prin

Check Your Progress Answers

1. B) To minimize the maximum loss
2. C) Decision-making under uncertainty with a focus on risk aversion
3. B) Maximin
4. B) The lowest possible loss for the player using the strategy
5. A) Minimax focuses on minimizing potential losses, while maximin focuses on maximizing potential gains.

UNIT IV

UNIT INRODUCTION

Inventory management is a critical component of operations research, dedicated to the systematic control and oversight of inventory to achieve optimal balance between cost efficiency and service level. This unit aims to delve into the core principles, theories, and practical applications of inventory management, equipping learners with the necessary tools and methodologies to streamline inventory processes, minimize associated costs, and enhance overall operational performance. Key topics covered include the various types of inventory, such as raw materials, work-in-progress, and finished goods, as well as the costs associated with holding, ordering, and shortages. Students will explore different inventory control systems like continuous review systems, periodic review systems, and Just-In-Time (JIT) inventory, alongside inventory management models such as the Economic Order Quantity (EOQ), Economic Production Quantity (EPQ), and reorder point (ROP) models. Additionally, the unit will address demand forecasting techniques and their impact on inventory management, as well as optimization methods including mathematical models, simulations, and heuristic approaches. Through a combination of lectures, readings, case studies, exercises, and hands-on software tool applications, students will gain a comprehensive understanding of effective inventory management strategies. Assessments will include quizzes, exams, assignments, and projects designed to test and apply the concepts learned. By the end of this unit, learners will be proficient in designing and implementing inventory management solutions that reduce costs and improve operational efficiency, providing a solid foundation for further studies and professional advancement in operations research.

4. 1 INTRODUCTION

The word ‘inventory’ means simply a stock of idle resources of any kind having an economic value. In other words, inventory means a physical stock of goods, which is kept in hand for smooth and efficient running of future affairs of an organization. It may consist of raw materials, work-in-progress, spare parts/consumables, finished goods, human resources such as unutilized labor, financial resources such as working capital, etc. It is not necessary that an organization has all these inventory classes but whatever may be the inventory items, they need efficient management as generally a substantial amount of money is invested in them. The basic inventory decisions in- clude:

1) *How much to order?*

2) *When to order?*

3) *How much safety stock should be kept?*

The problems faced by different organizations have necessitated the use of scientific techniques in the management of inventories known as inventory control. Inventory control is concerned with the acquisition, storage, and handling of inventories so that the inventory is available whenever needed and the associated total cost is minimized.

4.1.1 Reasons for Carrying Inventory

1. Improve customer service- An inventory policy is designed to respond to individual customer's or organization's request for products and services.

2.Reduce costs- Inventory holding or carrying costs are the expenses that are in- curred for storage of items. However, holding inventory items in the warehouse can indirectly reduce operating costs such as loss of goodwill and/or loss of po- tential sale due to shortage of items. It may also encourage economies of pro- duction by allowing larger, longer and more production runs.

3. Maintenance of operational capability- Inventories of raw materials and work- in-progress items act as buffer between successive production stages so that downtime in one stage does not affect the entire production process.

4.Irregular supply and demand- Inventories provide protection against irregular supply and demand; an unexpected change in production and delivery sched- ule of a product or a service can adversely affect operating costs and customer service level.

5.Quantity discount- Large size orders help to take advantage of price-quantity discount. However, such an advantage must keep a balance between the storage cost and costs due to obsolescence, damage, theft, insurance, etc.

6.Avoiding stockouts (shortages)- Under situations like, labor strikes, natural disasters, variations in demand and delays in supplies, etc., inventories act as buffer against stock out as well as loss of goodwill.

4.1.2 Costs Associated with Inventories

1. *Purchase (or production) cost*: It is the cost at which an item is purchased, or if an item is produced.
2. *Carrying (or holding) cost*: The cost associated with maintaining inventory is known as holding cost. It is directly proportional to the quantity kept in stock and the time for which an item is held in stock. It includes handling cost, main- tenance cost, depreciation, insurance, warehouse rent, taxes, etc.
3. *Shortage (or stock out) cost*: It is the cost which arises due to running out of stock. It includes the cost of production stoppage, loss of goodwill, loss of profitability, special orders at higher price, overtime/idle time payments, loss of opportunity to sell, etc.
4. *Ordering (or set up) cost*: The cost incurred in replenishing the inventory is known as ordering cost. It includes all the costs relating to administration (such as salaries of the persons working for purchasing, telephone calls, computer costs, postage, etc.), transportation, receiving and inspection of goods, processing pay- ments, etc. If a firm produces its own goods instead of purchasing the same from an outside source, then it is the cost of resetting the equipment for production.

4.1.3 Basic Terminologies

1.Demand

It is an effective desire which is related to particular time, price, and quantity. The demand pattern of a commodity may be either deterministic or probabilistic. In case of deterministic demand, the quantities needed in future are known with certainty. This can be fixed (static) or can vary (dynamic) from time to time. On the contrary, probabilistic demand is uncertain over a certain period of time but its pattern can be described by a known probability distribution.

2.Ordering cycle

An ordering cycle is defined as the time period between two successive replen- ishments. The order may be placed on the basis of the following two types of inventory review systems:

•Continuous review: In this case, the inventory level is monitored continu- ously until a specified point (known as reorder point) is reached. At this point, a new order is placed.

•Periodic review: In this case, the orders are placed at equally spaced intervals of time. The quantity ordered each time depends on the available inventory level at the time of review.

3.Planning period

This is also known as time horizon over which the inventory level is to be con- trolled. This can be finite or infinite depending on the nature of demand.

4.Lead time or delivery lag

The time gap between the moment of placing an order and actually receiving it is referred to as lead time. Lead time can be deterministic (constant or variable) or probabilistic.

5.Buffer (or safety) stock

Normally, demand and lead time are uncertain and cannot be predetermined completely. So, to absorb the variation in demand and supply, some extra stock is kept. This extra stock is known as buffer stock.

6. Re-order level

The level between maximum and minimum stocks at which purchasing activity must start for replenishment is known as re-order level.

4.1.4 CLASSIFICATIONS OF INVENTORY

Inventories can be classified into various categories based on their role in the production process, their physical state, or their purpose within a business. Here are the primary classifications of inventories along with suitable examples:

1. Raw Materials Inventory

Raw materials are the basic inputs that are used in the production process to manufacture finished goods. These are unprocessed materials that are converted into final products through manufacturing.

Example: Steel used in automobile manufacturing, cotton used in textile production, or flour used in baking.

2. Work-in-Progress (WIP) Inventory

Work-in-progress inventory consists of items that are in the process of being manufactured but are not yet complete. This includes raw materials, labor, and overhead costs that have been applied to the partially finished goods.

Example: Partially assembled cars on an automobile assembly line, dough rising in a bakery, or electronics being assembled in a factory.

3. Finished Goods Inventory

Finished goods are the final products that have completed the manufacturing process and are ready for sale to customers. These goods are waiting to be sold or shipped to customers.

Example: Completed vehicles ready for sale at a dealership, packaged bread in a bakery, or consumer electronics ready for distribution.

4. Maintenance, Repair, and Operations (MRO) Inventory

MRO inventory includes items that are used in the maintenance and repair of manufacturing equipment and operations but are not part of the final product. These are essential for keeping production processes running smoothly.

Example: Lubricants, cleaning supplies, spare parts for machinery, and tools.

5. Safety Stock Inventory

Safety stock is extra inventory held to mitigate the risk of stockouts caused by uncertainties in supply and demand. It acts as a buffer against unexpected demand spikes or supply chain disruptions.

Example: Extra pharmaceuticals in a hospital to cover unexpected patient needs, additional raw materials in a factory to account for supply delays, or extra consumer goods in a retail store for seasonal demand fluctuations.

6. Cycle Stock Inventory

Cycle stock is the portion of inventory that is expected to be used or sold within the normal operating cycle. It is the regular inventory required to meet expected demand over a specific period.

Example: Regular weekly supply of dairy products in a grocery store, monthly inventory of raw materials in a manufacturing plant, or daily stock of fresh produce in a supermarket.

7. Anticipation Inventory

Anticipation inventory is held in anticipation of future demand or price increases. Businesses stock up on these items to prepare for seasonal demand, promotions, or potential price hikes.

Example: Retailers stocking up on toys before the holiday season, manufacturers purchasing raw materials before a scheduled price increase, or wholesalers increasing inventory ahead of a major sales event.

8. Transit or Pipeline Inventory

Transit inventory refers to items that are in the process of being transported from one location to another. These items are on the way to their destination but have not yet been received.

Example: Goods being shipped from a supplier to a manufacturer, finished products en route from a factory to a distribution center, or raw materials being transported from a mining site to a processing plant.

9. Decoupling Inventory

Decoupling inventory is used to decouple different stages of production so that each stage can operate independently without disruptions from delays in other stages. It acts as a buffer between stages.

Example: Extra components stored between machining and assembly processes in a manufacturing plant, buffer stock of ingredients between preparation and cooking stages in a food production line, or intermediate products held between different phases of a chemical processing plant.

10. Obsolete or Dead Inventory

Obsolete inventory consists of items that are no longer in demand or useful. These items have surpassed their lifecycle and are unlikely to be sold or used in production.

Example: Outdated electronic components, fashion apparel from previous seasons that is no longer in style, or perishable goods that have expired.

Different classifications of inventories serve various roles within an organization. Effective inventory management ensures that the right type and amount of inventory are available to meet production schedules and customer demand while minimizing holding costs and avoiding stockouts. Understanding these classifications helps businesses optimize their inventory strategies and improve operational efficiency.

Let Us Sum Up

This gives a basic understanding on inventory meaning and cllasifications and its role in operations research.

Check Your Progress

1.Which of the following is NOT a type of inventory?

- A) Raw materials
- B) Work-in-progress
- C) Finished goods
- D) Capital equipment

2.The Economic Order Quantity (EOQ) model is primarily used to:

- A) Determine the optimal order quantity that minimizes total inventory costs
- B) Forecast future demand for products
- C) Calculate the reorder point for inventory
- D) Analyze the cost of stockouts

3.In a Just-In-Time (JIT) inventory system, the primary goal is to:

- A) Maintain high levels of safety stock
- B) Minimize inventory holding costs by receiving goods only as they are needed
- C) Maximize the lead time for orders
- D) Increase the frequency of orders to suppliers

4.Which inventory cost is associated with the storage and maintenance of inventory over time?

- A) Ordering cost

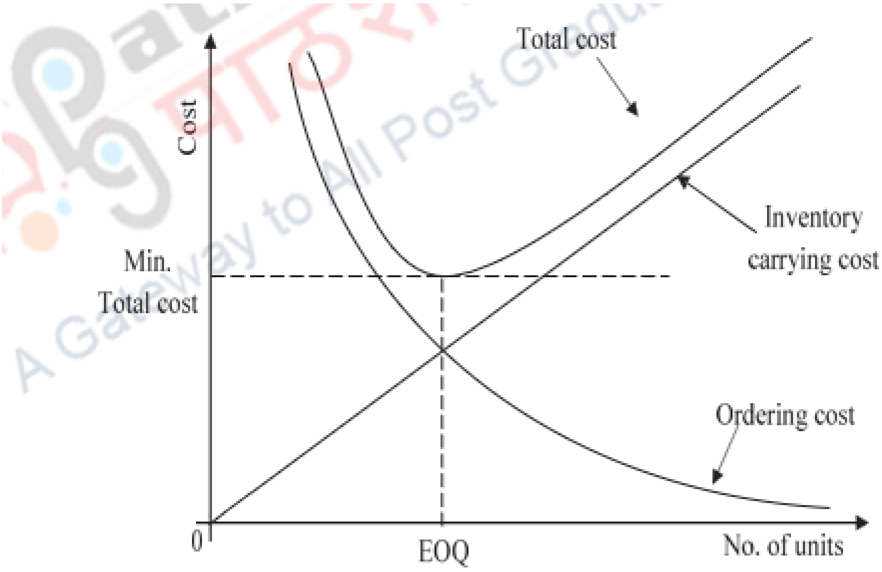
- B) Holding cost
- C) Shortage cost
- D) Setup cost

5. ABC analysis in inventory management is used to:

- A) Classify inventory items based on their importance and value
- B) Determine the optimal order quantity for each item
- C) Forecast future demand for inventory items
- D) Calculate the total cost of ownership for inventory items

4.5 ECONOMIC ORDER QUANTITY (EOQ)

The concept of economic ordering quantity (EOQ) was first developed by F. Harris in 1916. The concept is as follows: Management of inventory is confronted with a set of opposing costs. As the lot size increases, the carrying cost increases while the ordering cost decreases. On the other hand, as the lot size decreases, the carrying cost decreases but the ordering cost increases



flig. 1.1: Graph of EOQ

Economic ordering quantity (EOQ) is that size of order which minimizes the average total cost of carrying inventory and ordering under the assumed conditions of cer- tainty and the total demand during a given period of time is known.

4.6 SINGLE-PERIOD PROBABILISTIC INVENTORY MODELS

Single-period probabilistic inventory models, also known as newsvendor or newsboy problems, deal with situations where inventory must be ordered for a single period with uncertain demand. The goal is to find the optimal order quantity that maximizes expected profit or minimizes expected costs.

Discrete Demand

Steps to Determine Optimal Order Quantity for Discrete Demand:

1. **Define Parameters:**
    - DDD: Random variable representing demand.
    - QQQ: Order quantity.
    - CuC_uCu: Underage cost per unit (cost of lost sales).

   - CoC_oCo: Overage cost per unit (cost of holding unsold inventory).

2. **Determine Probability Distribution of Demand:**
   - Identify the probability P(D=di)P(D = d_i)P(D=di) for each possible demand did_idi.
3. **Calculate Cumulative Probability:**
   - Compute the cumulative probability distribution F(Q)F(Q)F(Q), where F(Q)=P(D≤Q)F(Q) = P(D \leq Q)F(Q)=P(D≤Q).
4. **Determine Optimal Order Quantity:**
   - Find QQQ such that P(D≤Q)≥CuCu+CoP(D \leq Q) \geq \frac{C_u}{C_u + C_o}P(D≤Q)≥Cu+CoCu.

Example:

- Demand can be 100, 200, or 300 units with probabilities 0.2, 0.5, and 0.3, respectively.
- Underage cost (CuC_uCu) = $2 per unit, overage cost (CoC_oCo) = $1 per unit.

CuCu+Co=22+1=23≈0.67\frac{C_u}{C_u + C_o} = \frac{2}{2 + 1} = \frac{2}{3} \approx 0.67Cu+CoCu=2+12=32≈0.67

Optimal QQQ corresponds to F(Q)≥0.67F(Q) \geq 0.67F(Q)≥0.67

D DDD	**Probability P(D=di)P d_i)P(D=di)**	**Cumulative F(Q)F(Q)F(Q)**	**Pro**
1	0.2	0.2	
2	0.5	0.7	
3	0.3	1.0	

Q=200Q = 200Q=200 is optimal because F(200)=0.7≥0.67F(200) = 0.7 \geq 0.67F(200)=0.7≥0.67.

4.7 CONTINUOUS DEMAND

Steps to Determine Optimal Order Quantity for Continuous Demand:

1.Define Parameters:

o DDD: Random variable representing demand, with probability density function f(d)f(d)f(d) and cumulative distribution function F(Q)F(Q)F(Q).

o QQQ: Order quantity.

o CuC_uCu: Underage cost per unit (cost of lost sales).

o CoC_oCo: Overage cost per unit (cost of holding unsold inventory).

2.Determine Optimal Order Quantity:

o Find QQQ such that F(Q)=CuCu+CoF(Q) = \frac{C_u}{C_u + C_o}F(Q)=Cu+CoCu.

Example:

• Demand follows a normal distribution with mean μ=500\mu = 500μ=500 and standard deviation σ=100\sigma = 100σ=100.

• Underage cost (CuC_uCu) = $2 per unit, overage cost (CoC_oCo) = $1 per unit.

CuCu+Co=22+1=23≈0.67\frac{C_u}{C_u + C_o} = \frac{2}{2 + 1} = \frac{2}{3} \approx 0.67Cu+CoCu=2+12=32≈0.67

• Find QQQ such that F(Q)=0.67F(Q) = 0.67F(Q)=0.67.

Using the standard normal distribution table, Z0.67≈0.44Z_{0.67} \approx 0.44Z0.67≈0.44.

Q=μ+Zσ=500+0.44×100=544Q = \mu + Z \sigma = 500 + 0.44 \times 100 = 544Q=μ+Zσ=500+0.44×100=544

Determination of Reorder Point

Deterministic Inventory System

1.Define Parameters:

o DDD: Demand rate.

o LLL: Lead time.

o ROPROPROP: Reorder point.

2.Calculate Reorder Point:

o ROP=D×LROP = D \times LROP=D×L

Example:

• Demand rate (DDD) = 50 units per week.

• Lead time (LLL) = 2 weeks.

ROP=50×2=100 unitsROP = 50 \times 2 = 100 \text{ units}ROP=50×2=100 units

Probabilistic Inventory System

1.Define Parameters:

o DDD: Average demand rate.

o LLL: Lead time.

o σD\sigma_DσD: Standard deviation of demand.

o σL\sigma_LσL: Standard deviation of lead time.

o zzz: Safety factor corresponding to desired service level.

o ROPROPROP: Reorder point.

2.Calculate Safety Stock:

o If demand and lead time are independent: σDL=LσD2+D2σL2\sigma_{DL} = \sqrt{L \sigma_D^2 + D^2 \sigma_L^2}σDL=LσD2+D2σL2

o Safety stock SS=zσDLSS = z \sigma_{DL}SS=zσDL.

3.Calculate Reorder Point:

o ROP=D×L+SSROP = D \times L + SSROP=D×L+SS

Example:

• Average demand (DDD) = 50 units per week.

• Lead time (LLL) = 2 weeks.

• Standard deviation of demand (σD\sigma_DσD) = 10 units per week.

• Standard deviation of lead time (σL\sigma_LσL) = 0.5 weeks.

• Desired service level corresponds to z=1.65z = 1.65z=1.65 (for approximately 95% service level).

$$\sigma_{DL} = \sqrt{2 \times 10^2 + (50^2 \times 0.5^2)} = \sqrt{200 + 625} = \sqrt{825} \approx 28.72$$

$$SS = 1.65 \times 28.72 \approx 47.38$$

$$ROP = 50 \times 2 + 47.38 = 100 + 47.38 \approx 147 \text{ units}$$

These steps can be used to solve various inventory problems involving single-period probabilistic models and reorder point calculations for both deterministic and probabilistic inventory systems

4.8 JUST-IN-TIME (JIT)

Just-In-Time (JIT) is an inventory management and production strategy aimed at increasing efficiency and reducing waste by receiving goods only as they are needed in the production process, thereby minimizing inventory costs. Here are the basic concepts of JIT:

1. **Demand-Pull System:**
   - JIT operates on a demand-pull basis, where production is driven by actual customer demand rather than forecasted demand. This reduces overproduction and excess inventory.

2. **Elimination of Waste:**
   - One of the core principles of JIT is the elimination of waste (muda). This includes waste of time, materials, and labor. The goal is to streamline processes and eliminate any activities that do not add value to the final product.
3. **Continuous Improvement (Kaizen):**
   - JIT emphasizes continuous improvement. Employees at all levels are encouraged to identify areas for improvement and make incremental changes to enhance efficiency and productivity.
4. **Quality Control:**
   - High-quality standards are crucial in JIT since there is minimal inventory to buffer against defects. This requires rigorous quality control at every stage of production to ensure that defects are caught early and corrected promptly.
5. **Supplier Relationships:**
   - JIT relies on strong relationships with suppliers to ensure timely delivery of materials and components. Suppliers must be reliable and able to deliver small quantities frequently to align with production schedules.
6. **Reduction of Lead Time:**
   - Shortening lead times is essential in JIT to respond quickly to customer demands. This involves optimizing processes, improving communication with suppliers, and reducing setup times.
7. **Production Smoothing (Heijunka):**
   - Production smoothing, or leveling (heijunka), involves balancing production to avoid bottlenecks and fluctuations. This ensures a steady flow of work and reduces the variability in production.
8. **Small Lot Sizes:**
   - JIT encourages producing in small lot sizes to reduce inventory holding costs and improve responsiveness. Smaller lot sizes also help identify and solve problems more quickly.

9. **Kanban System:**
    - The Kanban system is a visual tool used in JIT to signal the need for inventory replenishment. It helps manage workflow and ensures that materials are available when needed without excess inventory.
10. **Flexible Workforce:**
    - A flexible workforce is vital in JIT. Employees are often cross-trained to perform multiple tasks, allowing for flexibility in production and quick adaptation to changing demands.
11. **Cellular Manufacturing:**
    - Cellular manufacturing involves arranging production workstations in a sequence that supports a smooth flow of materials and components. This setup reduces movement and handling, enhancing efficiency.
12. **Total Productive Maintenance (TPM):**
    - TPM is a proactive approach to maintenance that aims to prevent equipment breakdowns and ensure that machinery operates efficiently. Regular maintenance schedules and employee involvement in maintenance activities are key components.

4.8.1 Benefits of JIT:

- **Reduced Inventory Costs:** Lower inventory levels reduce holding costs and free up capital.
- **Increased Efficiency:** Streamlined processes and reduced waste lead to higher productivity.
- **Improved Quality:** Continuous improvement and quality control lead to fewer defects and higher-quality products.
- **Enhanced Customer Satisfaction:** Faster response to customer demands and shorter lead times improve service levels.

4.8.2 Challenges of JIT:

- **Supplier Dependence:** JIT requires reliable suppliers who can deliver on time.

- **Vulnerability to Disruptions:** Any delay in the supply chain can halt production, making JIT systems sensitive to disruptions.
- **Initial Implementation Costs:** Transitioning to JIT can involve significant changes to processes and may require investment in training and new systems.

4.9 MATERIALS REQUIREMENT PLANNING (MRP)

Materials Requirement Planning (MRP) is a production planning, scheduling, and inventory control system used to manage manufacturing processes. Its primary objective is to ensure that materials are available for production and products are available for delivery to customers, while maintaining the lowest possible level of inventory. MRP, which is done primarily through specialized software, helps ensure that the right inventory is available for the production process exactly when it is needed and at the lowest possible cost. As such, MRP improves the efficiency, flexibility and profitability of manufacturing operations. It can make factory workers more productive, improve product quality and minimize material and labor costs. MRP also helps manufacturers respond more quickly to increased demand for their products and avoid production delays and inventory stockouts that can result in lost customers, which in turn contributes to revenue growth and stability.

MRP is widely used by manufacturers and has undeniably been one of the key enablers in the growth and wide availability of affordable consumer goods and, consequently, has raised the standard of living in most countries. Without a way to automate the complex calculations and data management of MRP processes, it is unlikely that individual manufacturers could have scaled up operations as rapidly as they have in the half century since MRP software arrived.

Here are the key aspects of MRP:

4.9.1 Key Components of MRP:

1. Master Production Schedule (MPS):
   - A plan for the production of finished goods.
   - Specifies what is to be made, in what quantities, and when.
2. Bill of Materials (BOM):
   - A comprehensive list of raw materials, components, and assemblies needed to manufacture a product.
   - Structured as a hierarchy of components, subassemblies, and raw materials.
3. Inventory Records:
   - Detailed information on the current inventory status of each item.
   - Includes data on on-hand quantities, scheduled receipts, and lead times.

How does MRP work

MRP uses information from the bill of materials (BOM), inventory data and the master production schedule to calculate the required materials and when they will be needed in the manufacturing process. The BOM is a hierarchical list of all the materials, subassemblies and other components needed to make a product, along with their quantities, each usually shown in a parent-child relationship. The finished good is the parent at the top of the hierarchy.

The inventory items in the BOM are classified as either *independent demand* or *dependent demand*. An independent demand item is the finished good at the top of the hierarchy. Manufacturers determine its amount by considering confirmed orders and examining market conditions, past sales and other indicators to create a forecast, then decide how many to make to meet the expected demand.

Dependent demand items, in contrast, are the raw materials and components needed to make the finished product. For each of these items, demand depends on how many are needed to make the next-highest component in the BOM hierarchy.

MRP is the system most companies use to track and manage all of these dependencies and to calculate the number of items needed by the dates specified in the master production schedule. To put it another way, MRP is an inventory management and control system for ordering and tracking the items needed to make a product.

Lead time -- the period from when an order is placed and the item delivered -- is another key concept in MRP. There are many types of lead times. Two of the most common are material lead time (the time it takes to order materials and receive them) and factory or production lead time (how long it takes to make and ship the product after all materials are in). Customer lead time denotes the time between the customer order and final delivery. The MRP system calculates many of these lead times, but some are chosen by the operations managers and entered manually.

4.9.2 Core Functions of MRP:

1. Demand Forecasting:
    - Estimates future demand for products.
    - Used to create the Master Production Schedule.
2. Exploding the BOM:
    - Breaks down the BOM to determine the quantity of each component and raw material required to meet the production schedule.
3. Inventory Management:
    - Tracks inventory levels to ensure that there are enough materials to meet production needs without excessive overstocking.
4. Scheduling:
    - Plans the timing for ordering and receiving materials.
    - Coordinates the production schedule with material availability.
5. Order Management:
    - Generates purchase orders for raw materials and components.
    - Schedules production orders for manufacturing processes.

4.9.3 Benefits of MRP

1. Improved Inventory Control:
    - Helps maintain optimal inventory levels, reducing carrying costs and minimizing stockouts.
2. Enhanced Production Planning:
    - Ensures materials are available when needed, improving production efficiency and reducing lead times.
3. Better Customer Service:
    - Increases the ability to meet customer demand on time by ensuring product availability.
4. Cost Reduction:
    - Minimizes waste and inefficiencies in the production process, leading to cost savings.
5. Reduced customer lead times to improve customer satisfaction.
6. Reduced inventory costs.
7. Effective inventory management and optimization -- by acquiring or manufacturing the optimal amount and type of inventory, companies can minimize the risk of stock-outs, and their negative impact on customer satisfaction, sales and revenue, without spending more than necessary on inventory.
8. Improved manufacturing efficiency by using accurate production planning and scheduling to optimize the use of labor and equipment.
9. Improved labor productivity.

4.9.4 Limitations of MRP:

1. Complexity:
    - Requires accurate data and can be complex to implement and manage.

2. Dependency on Accurate Data:
   - Inaccurate data on inventory levels, lead times, and demand forecasts can lead to errors in planning and scheduling.
3. Static Planning:
   - Traditional MRP systems can struggle to adapt quickly to changes in demand or production conditions.
4. Oversupply of inventory. While MRP is designed to ensure adequate inventory levels at the required times, companies can be tempted to hold more inventory than is necessary, thereby driving up inventory costs. An MRP system anticipates shortages sooner, which can lead to overestimating inventory lot sizes and lead times, especially in the early days of deployment before users gain the experience to know the actual amounts needed.
5. Lack of flexibility. MRP is also somewhat rigid and simplistic in how it accounts for lead times or details that affect the master production schedule, such as the efficiency of factory workers or issues that can delay delivery of materials.
6. Data integrity requirements. MRP is highly dependent on having accurate information about key inputs, especially demand, inventory and production. If one or two inputs are inaccurate, errors can be magnified at later stages. Data integrity and data management are thus essential to effective use of MRP systems.

To address these shortcomings of MRP, many manufacturers use advanced planning and scheduling (APS) software, which uses sophisticated math and logic to provide more accurate and realistic estimates of lead times. Unlike most MRP systems, APS software accounts for production capacity, which can have a significant impact on availability of materials.

4.9.5 Modern Developments

1. MRP II (Manufacturing Resource Planning):
   - An extension of MRP that integrates additional data, such as labor and machine capacity, to provide a more comprehensive planning approach.
2. ERP (Enterprise Resource Planning):

   - Integrates MRP with other business processes like finance, human resources, and customer relationship management, providing a holistic view of the business.

4.9.6 Implementation Steps

1. Define Requirements:
   - Determine the scope and objectives of the MRP system.
2. Data Collection:
   - Gather accurate data on inventory, BOMs, lead times, and demand forecasts.
3. System Selection:
   - Choose an appropriate MRP software system that meets the organization's needs.
4. Training and Testing:
   - Train staff on the new system and conduct thorough testing to ensure it functions correctly.
5. Go-Live and Monitoring:
   - Implement the system and continuously monitor its performance, making adjustments as necessary.

MRP is a critical tool for managing manufacturing processes, ensuring materials are available for production, and products are available for delivery while minimizing inventory levels. It requires careful planning, accurate data, and continuous monitoring to be effective. Modern advancements like MRP II and ERP systems have enhanced the capabilities of traditional MRP, providing more comprehensive and integrated planning solutions.

Let us Sum Up

The Economic Order Quantity (EOQ) and Just-In-Time (JIT) are two fundamental inventory management approaches in operations research, each with distinct objectives and applications. The EOQ model focuses on determining the optimal order quantity that minimizes the total cost of inventory, which includes both ordering costs and holding costs. By calculating the ideal order size, EOQ helps businesses reduce overall inventory expenses and maintain a balance between ordering frequency and inventory levels. On the other hand, JIT inventory management aims to minimize holding costs by aligning inventory orders closely with production schedules and customer demand. This approach reduces the amount of inventory on hand, thereby cutting storage costs and minimizing waste. While EOQ is advantageous for maintaining a stable inventory level with predictable demand, JIT is beneficial for environments where flexibility and responsiveness to market changes are critical. Both methods, when applied appropriately, can significantly enhance operational efficiency and cost-effectiveness in inventory management.

Check Your Progress

1. The primary objective of the Economic Order Quantity (EOQ) model is to:
   - A) Maximize the frequency of orders
   - B) Minimize the total inventory costs
   - C) Reduce lead time for orders
   - D) Increase the amount of safety stock
2. In the EOQ formula, the variable "D" typically represents:
   - A) The demand rate or annual usage of the item
   - B) The cost per order
   - C) The holding cost per unit per year
   - D) The reorder point
3. A Just-In-Time (JIT) inventory system aims to:
   - A) Maintain large amounts of inventory to meet unexpected demand
   - B) Align inventory orders with production schedules and reduce inventory holding
   - C) Increase the lead time for suppliers
   - D) Focus on ordering in bulk to minimize ordering costs
4. Which of the following is a key benefit of implementing a JIT inventory system?
   - A) High levels of safety stock

- B) Reduced inventory holding costs and minimized waste
- C) Increased complexity in managing inventory
- D) Higher ordering costs due to frequent orders

5. In the EOQ model, the trade-off is primarily between:
    - A) Holding costs and setup costs
    - B) Ordering costs and holding costs
    - C) Shortage costs and setup costs
    - D) Demand variability and lead time

UNIT SUMMARY

Inventory management in operations research focuses on optimizing the control and oversight of inventory to balance cost efficiency and service levels. This unit covers essential concepts such as the types of inventory—raw materials, work-in-progress, finished goods, and MRO supplies—and the associated costs, including holding, ordering, shortage, and setup costs. Various inventory control systems are explored, such as continuous review (Q-system), periodic review (P-system), and Just-In-Time (JIT) inventory systems, each with its benefits and limitations. Key inventory management models, including the Economic Order Quantity (EOQ), Economic Production Quantity (EPQ), and reorder point (ROP) models, are discussed to help determine optimal order quantities and timing. The unit also delves into demand forecasting techniques and their impact on inventory management, along with optimization methods like mathematical models, simulations, and heuristic approaches. Practical applications are illustrated through real-world case studies and hands-on experience with inventory management software tools. This comprehensive exploration equips students with the knowledge and skills to effectively manage inventory, reduce costs, and improve operational efficiency.

Glossary

1. **Economic Order Quantity (EOQ)**: A formula used to determine the optimal order quantity that minimizes the total inventory costs, including ordering and holding costs. It helps in balancing the cost trade-offs and determining the most cost-effective order size.

2. **Reorder Point (ROP)**: The inventory level at which a new order should be placed to replenish stock before it runs out. It considers the lead time and the rate of demand to ensure that there is enough inventory to meet customer demand until the new order arrives.
3. **Just-in-Time (JIT)**: An inventory management strategy aimed at reducing inventory holding costs by receiving goods only as they are needed in the production process. This approach minimizes waste and improves efficiency by reducing the amount of inventory kept on hand.
4. **Safety Stock**: Extra inventory kept on hand to protect against uncertainties in demand or supply. It acts as a buffer to ensure that there are sufficient goods available to meet unexpected increases in demand or delays in supply.
5. **Material Requirement Planning (MRP)**: A system used for planning and controlling inventory, production, and scheduling. It ensures that materials are available for production and products are available for delivery to customers, while maintaining the lowest possible level of inventory.

Self-Assesment Questions

1. What are the different types of inventory systems, and how are they classified?

2. Explain the economic order quantity (EOQ) model and its significance in inventory management.

3. How does the EOQ model help in minimizing inventory costs?

4. What factors influence the determination of the EOQ?

6. Describe the assumptions underlying the EOQ model.
7. What is the difference between deterministic and probabilistic inventory models?
8. Discuss the single-period probabilistic inventory model with discrete demand.
9. How is the reorder point determined in a deterministic inventory system?
10. Explain the concept of safety stock and its role in inventory management.
11. What are the challenges associated with managing inventory in a probabilistic environment?

12. Describe the continuous demand model in inventory management.
13. What is the significance of lead time in inventory management?
14. Discuss the concept of Just-in-Time (JIT) inventory management.
15. How does JIT differ from traditional inventory management systems?
16. Explain the benefits and drawbacks of implementing JIT in a manufacturing environment.
17. What is Material Requirement Planning (MRP), and how does it contribute to inventory control?
18. Describe the basic components of an MRP system.
19. How does MRP help in optimizing inventory levels and production scheduling?
20. Discuss the importance of accurate demand forecasting in inventory management.
21. What are some of the key performance indicators (KPIs) used to evaluate inventory management effectiveness?

Exercise

22. company sells 5,000 units of a product annually. The cost of placing an order is $50, and the carrying cost per unit per year is $2. Calculate the Economic Order Quantity (EOQ).
23. A store sells 10,000 units of a product each year. The ordering cost is $100 per order, and the annual carrying cost per unit is $5. Calculate the EOQ for this product.A manufacturing company has an annual demand of 20,000 units for a component. The ordering cost is $200 per order, and the carrying cost per unit per year is
24. A retailer sells 15,000 units of a product annually. The cost of placing an order is $80, and the carrying cost per unit per year is $4. Calculate the EOQ for this product.
25. A warehouse orders 25,000 units of a particular item each year. The ordering cost is $150 per order, and the carrying cost per unit per year is $6. Calculate the EOQ for this item.

Check Your Progress Answers

1.

D) Capital equipment

A) Determine the optimal order quantity that minimizes total inventory costs

B) Minimize inventory holding costs by receiving goods only as they are needed

B) Holding cost

A) Classify inventory items based on their importance and value

2

B) Minimize the total inventory costs

A) The demand rate or annual usage of the item

B) Align inventory orders with production schedules and reduce inventory holding

B) Reduced inventory holding costs and minimized waste

B) Ordering costs and holding costs

UNIT V

UNIT INTRODUCTION

Network analysis in operations research is a critical field that focuses on the optimization of complex networks within various operational contexts. This unit introduces the fundamental concepts and methodologies of network analysis, emphasizing its application in solving real-world problems related to logistics, transportation, supply chain management, and project scheduling. Students will explore key topics such as network flow models, shortest path algorithms, maximum flow problems, and project management techniques like the Critical Path Method (CPM) and Program Evaluation and Review Technique (PERT). Through a combination of theoretical frameworks and practical applications, this unit aims to equip learners with the analytical tools necessary to design, analyze, and optimize networks, ultimately enhancing decision-making and operational efficiency in diverse industries.

5.NETWORK ANALYSIS

1.Defining the job to be done

2.Integrating the elements of the job in a logical time sequence

3.Controlling the progress of the project.

Network analysis is concerned with minimizing some measure of performance of the system such as the total completion time for the project, overall cost and so on. By preparing a network of the system, a decision maker can identify,

(i)The physical relationship (properties) of the system

(ii)The inter relationships of the system components

Objectives:

Network analysis can be used to serve the following objectives:

1.Minimization of total time: Network analysis is useful in completing a project in the minimum possible time. A good example of this objective is the maintenance of production line machinery in a factory. If the cost of down time is very high, it is economically desirable to minimize time despite high resource costs.

2.Minimization of total cost: Where the cost of delay in the completion of the project exceeds cost of extra effort, it is desirable to complete the project in time so as to minimize total cost.

3.Minimization of time for a given cost: When fixed sum is available to cover costs, it may be preferable to arrange the existing resources so as to reduce the total time for the project instead of reducing total cost.

4.Minimization of cost for a given total time: When no particular benefit will be gained from completing the project early, it may be desirable to arrange resources in such a way as to give the minimum cost for the project in the set time.

5.Minimization of idle resources: The schedule should be devised to minimize large fluctuations in the use of limited resources. The cost of having men/machines idle should be compared with the cost of hiring resources on a temporary basis.

6.Network analysis can also be employed to minimize production delays, interruptions and conflicts.

5.1 Managerial Applications

Network analysis can be applied to very wide range of situations involving the use of time, labour and physical resources. Some of the more common applications of network analysis in project scheduling are as follows:

1. Construction of bridge, highway, power plant etc.
2. Assembly line scheduling.
3. Installation of a complex new equipment. Eg. Computers, large machinery.
4. Research and Development
5. Maintenance and overhauling complicated equipment in chemical or power plants, steel and petroleum industries, etc.
6. Inventory planning and control.
7. Shifting of manufacturing plant from one site to another.
8. Development and testing of missile system.
9. Development and launching of new products and advertising campaigns.

10. Repair and maintenance of an oil refinery.
11. Construction of residential complex.
12. Control of traffic flow in metropolitan cities.
13. Long range planning and developing staffing plans.
14. Budget and audit procedures.
15. Organization of international conferences.
16. Launching space programmes, etc.

A network is a graphic representation of a project's operations and is composed of activities and events (or nodes) that must be completed to reach the end objective of a project, showing the planning sequence of their accomplishments, their dependence and inter relationships.

5.1.1 Network Diagram Representation

In a network representation of a project certain definitions are used

Activity

Any individual operation which utilizes resources and has an end and a beginning is called activity.

An arrow is commonly used to represent an activity with its head indicating the direction of progress in the project. These are classified into four categories

1.Predecessor activity – Activities that must be completed immediately prior to the start of another activity are called predecessor activities.

2.Successor activity – Activities that cannot be started until one or more of other activities are completed but immediately succeed them are called successor activities.

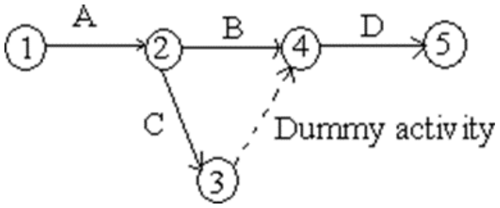
3.Concurrent activity – Activities which can be accomplished concurrently are known as concurrent activities. It may be noted that an activity can be a predecessor or a successor to an event or it may be concurrent with one or more of other activities.

4.Dummy activity – An activity which does not consume any kind of resource but merely depicts the technological dependence is called a dummy activity.

The dummy activity is inserted in the network to clarify the activity pattern in the following two situations

- •To make activities with common starting and finishing points distinguishable
- •To identify and maintain the proper precedence relationship between activities that is not connected by events.

For example, consider a situation where A and B are concurrent activities. C is dependent on A and D is dependent on A and B both. Such a situation can be handled by using a dummy activity as shown in the figure.

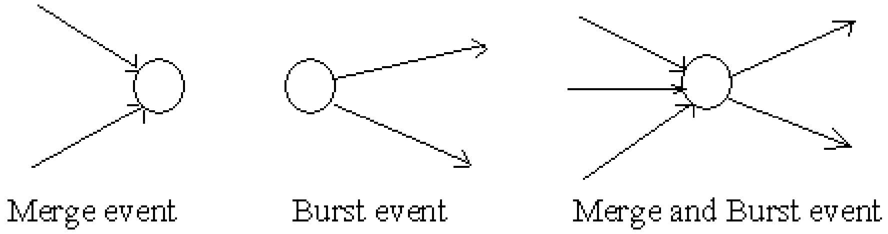


Event

An event represents a point in time signifying the completion of some activities and the beginning of new ones. This is usually represented by a circle in a network which is also called a node or connector.

The events are classified in to three categories

1. Merge event – When more than one activity comes and joins an event such an event is known as merge event.
2. Burst event – When more than one activity leaves an event such an event is known as burst event.
3. Merge and Burst event – An activity may be merge and burst event at the same time as with respect to some activities it can be a merge event and with respect to some other activities it may be a burst event.



3. Sequencing

The first prerequisite in the development of network is to maintain the precedence relationships. In order to make a network, the following points should be taken into considerations

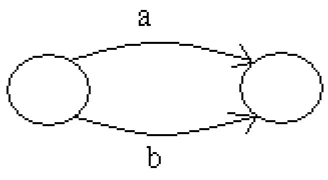
- What job or jobs precede it?
- What job or jobs could run concurrently?
- What job or jobs follow it?
- What controls the start and finish of a job?

Since all further calculations are based on the network, it is necessary that a network be drawn with full care.

5.1.2 Rules for Drawing Network Diagram

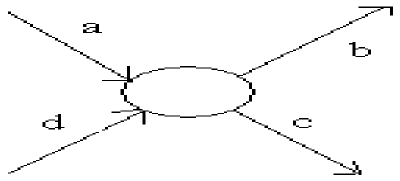
Rule 1

Each activity is represented by one and only one arrow in the network



Rule 2

No two activities can be identified by the same end events



Rule 3

In order to ensure the correct precedence relationship in the arrow diagram, following questions must be

checked whenever any activity is added to the network

- What activity must be completed immediately before this activity can start?
- What activities must follow this activity?
- What activities must occur simultaneously with this activity?

In case of large network, it is essential that certain good habits be practiced to draw an easy to follow Network

- Try to avoid arrows which cross each other
- Use straight arrows
- Do not attempt to represent duration of activity by its arrow length
- Use arrows from left to right. Avoid mixing two directions, vertical and standing arrows may be used if necessary.
- Use dummies freely in rough draft but final network should not have any redundant dummies.
- The network has only one entry point called start event and one point of emergence called the end event.

5.2 CPM (CRITICAL PATH METHOD) AND PERT (PROGRAM EVALUATION AND REVIEW TECHNIQUE)

Both CPM and PERT are project management techniques used to plan, schedule, and control complex projects. They are particularly useful in identifying the sequence of critical tasks that determine the project's duration and ensuring that projects are completed on time.

5.2.1 Critical Path Method (CPM)

Overview

CPM is a deterministic project management technique used for scheduling activities within a project. It focuses on the critical path, the longest path through the project's activity network, which determines the minimum project duration. CPM is best suited for projects with well-defined tasks and durations.

Steps to Determine the Critical Path Using CPM

1. **Define the Project and List Activities:**
   - Break down the project into smaller tasks or activities.
   - Identify dependencies between activities (i.e., which tasks must be completed before others can start).
2. **Develop a Network Diagram:**
   - Create a visual representation of the activities and their dependencies.
   - Use nodes to represent activities and directed arrows to indicate the sequence.
3. **Estimate Activity Durations:**
   - Assign a specific duration to each activity based on experience, historical data, or expert judgment.
4. **Identify Paths and Calculate Path Durations:**
   - Determine all possible paths from the project start to the project end.
   - Calculate the total duration for each path by summing the durations of all activities on the path.
5. **Determine the Critical Path:**
   - Identify the longest path through the network diagram. This is the critical path.
   - The duration of the critical path is the shortest possible time to complete the project.

6. **Calculate Early and Late Start/Finish Times:**
   - Perform a forward pass to calculate the earliest start (ES) and finish (EF) times for each activity.
   - Perform a backward pass to calculate the latest start (LS) and finish (LF) times for each activity.
7. **Determine Slack Time:**
   - Calculate slack time (or float) for each activity using the formula: Slack=LS−ES
8. **Activities on the critical path have zero slack.**

Example

Activity	Duration (days)	Predecessors
A	3	-
B	4	A
C	2	A
D	5	B, C
E	6	C
F	3	D, E

Steps:

1. Develop a Network Diagram:

A --> B --> D --> F

\ /

--> C --> E

1. **Identify Paths and Calculate Durations:**

- Path 1: A-B-D-F = 3 + 4 + 5 + 3 = 15 days
- Path 2: A-C-D-F = 3 + 2 + 5 + 3 = 13 days
- Path 3: A-C-E-F = 3 + 2 + 6 + 3 = 14 days

2. **Critical Path:** Path 1 (A-B-D-F) with duration 15 days.
3. **Forward and Backward Pass to Determine Early/Late Start/Finish Times:**
   - Activity A: ES = 0, EF = 3
   - Activity B: ES = 3, EF = 7
   - Activity C: ES = 3, EF = 5
   - Activity D: ES = 7, EF = 12
   - Activity E: ES = 5, EF = 11
   - Activity F: ES = 12, EF = 15

Let Us Sum Up

This unit introduces the fundamental concepts and methodologies of network analysis, emphasizing its application in solving real-world problems related to logistics, transportation, supply chain management, and project scheduling. Students will explore key topics such as network flow models, shortest path algorithms, maximum flow problems, and project management techniques like the Critical Path Method (CPM) and Program Evaluation and Review Technique (PERT)

Check Your Progress

1. Which algorithm is commonly used to find the shortest path in a network?
   - A) Dijkstra's algorithm
   - B) Ford-Fulkerson algorithm
   - C) Kruskal's algorithm
   - D) Prim's algorithm
2. In a network flow problem, the maximum flow from a source to a sink can be found using:

- A) Bellman-Ford algorithm
- B) Ford-Fulkerson algorithm
- C) Floyd-Warshall algorithm
- D) Kruskal's algorithm

3. The Critical Path Method (CPM) is primarily used for:
    - A) Finding the shortest path in a network
    - B) Determining the maximum flow in a network
    - C) Scheduling and managing complex projects
    - D) Minimizing the cost of network connections
4. In project management, the critical path is defined as:
    - A) The path with the maximum number of activities
    - B) The longest path through the network with the shortest completion time
    - C) The path with the least number of dependencies
    - D) The shortest path from the start to the finish of the project
5. PERT (Program Evaluation and Review Technique) is used to:
    - A) Calculate the shortest path in a transportation network
    - B) Optimize resource allocation in a supply chain
    - C) Estimate the project duration considering uncertainty
    - D) Maximize the flow in a network

5.2.2 PROGRAM EVALUATION AND REVIEW TECHNIQUE (PERT)

Overview

PERT is a probabilistic project management technique used to estimate the duration of activities where there is uncertainty. It involves three time estimates for each activity: optimistic (O), most likely (M), and pessimistic (P). PERT is best suited for research and development projects with high uncertainty.

Steps to Determine the Critical Path Using PERT

1. **Define the Project and List Activities:**
   - Similar to CPM, break down the project into smaller tasks or activities.
   - Identify dependencies between activities.
2. **Develop a Network Diagram:**
   - Similar to CPM, create a visual representation of activities and their dependencies.
3. **Estimate Activity Durations:**
   - Use three-time estimates for each activity: optimistic (O), most likely (M), and pessimistic (P).
   - Calculate the expected duration (TE) for each activity using the formula: TE=O+4M+P6TE = \frac{O + 4M + P}{6}TE=6O+4M+P
4. **Calculate Variance for Each Activity:**
   - Calculate the variance (σ2\sigma^2σ2) for each activity using the formula: σ2=(P−O6)2\sigma^2 = \left(\frac{P - O}{6}\right)^2σ2=(6P−O)2
5. **Identify Paths and Calculate Path Durations:**
   - Determine all possible paths from the project start to the project end.
   - Calculate the total expected duration for each path by summing the expected durations (TE) of all activities on the path.
6. **Determine the Critical Path:**

- Identify the longest path through the network diagram based on the expected durations (TE).
- Calculate the project duration using the expected duration of the critical path.

7. **Calculate Project Duration Variance:**
   - Sum the variances of activities on the critical path to find the project duration variance (σproject2\sigma^2_{project}σproject2).
   - Calculate the project standard deviation (σproject\sigma_{project}σproject) as: σproject=σproject2\sigma_{project} = \sqrt{\sigma^2_{project}}σproject =σproject2

8. **Determine Probability of Meeting Deadlines:**
   - Use the project duration and standard deviation to determine the probability of completing the project by a certain time using the standard normal distribution.

Example

Activity	Optimistic (O)	Most Likely (M)	Pessimistic (P)	Predecessors
A	2	3	4	-
B	3	4	5	A
C	1	2	3	A
D	4	5	6	B, C
E	5	6	7	C
F	2	3	4	D, E

Steps:

1. **Calculate Expected Durations (TE):**
    - Activity A: TE=2+4(3)+46=3TE = \frac{2 + 4(3) + 4}{6} = 3TE=62+4(3)+4=3 days
    - Activity B: TE=3+4(4)+56=4TE = \frac{3 + 4(4) + 5}{6} = 4TE=63+4(4)+5=4 days
    - Activity C: TE=1+4(2)+36=2TE = \frac{1 + 4(2) + 3}{6} = 2TE=61+4(2)+3=2 days
    - Activity D: TE=4+4(5)+66=5TE = \frac{4 + 4(5) + 6}{6} = 5TE=64+4(5)+6=5 days
    - Activity E: TE=5+4(6)+76=6TE = \frac{5 + 4(6) + 7}{6} = 6TE=65+4(6)+7=6 days
    - Activity F: TE=2+4(3)+46=3TE = \frac{2 + 4(3) + 4}{6} = 3TE=62+4(3)+4=3 days
2. **Calculate Variance (σ2\sigma^2σ2):**
    - Activity A: σ2=(4−26)2=0.111\sigma^2 = \left(\frac{4 - 2}{6}\right)^2 = 0.111σ2=(64−2)2=0.111
    - Activity B: σ2=(5−36)2=0.111\sigma^2 = \left(\frac{5 - 3}{6}\right)^2 = 0.111σ2=(65−3)2=0.111
    - Activity C: σ2=(3−16)2=0.111\sigma^2 = \left(\frac{3 - 1}{6}\right)^2 = 0.111σ2=(63−1)2=0.111
    - Activity D: σ2=(6−46)2=0.111\sigma^2 = \left(\frac{6 - 4}{6}\right)^2 = 0.111σ2=(66−4)2=0.111
    - Activity E: σ2=(7−56)2=0.111\sigma^2 = \left(\frac{7 - 5}{6}\right)^2 = 0.111σ2=(67−5)2=0.111
    - Activity F: σ2=(4−26)2=0.111\sigma^2 = \left(\frac{4 - 2}{6}\right)^2 = 0.111σ2=(64−2)2=0.111
3. **Identify Paths and Calculate Expected Durations:**
    - Path 1: A-B-D-F = 3 + 4 + 5 + 3 = 15 days
    - Path 2: A-C-D-F = 3 + 2 + 5 + 3 = 13 days
    - Path 3: A-C-E-F = 3 + 2 + 6 + 3 = 14 days
4. **Critical Path:** Path 1 (A-B-D-F) with an expected duration of 15 days.

5. **Calculate Project Duration Variance and Standard Deviation:**
    - Variance for Path 1: (0.111 + 0.111 + 0.111 + 0.111 = 0.

5.2.3 PERT crashing a project

In PERT (Program Evaluation and Review Technique), crashing a project refers to the process of reducing the project's total duration by expediting critical activities, typically at an additional cost. This approach is used when there is a need to meet a tight deadline or when project completion time is critical.

Steps to Crashing a Project in PERT:

1. **Identify Critical Activities:**
    - Determine the critical path in the project using PERT analysis.
    - Critical activities are those on the critical path, meaning any delay in these activities will delay the entire project.
2. **Determine Normal and Crash Times:**
    - Normal Time (TN): The time required to complete an activity under normal conditions.
    - Crash Time (TC): The shortest possible time to complete an activity by expediting resources or adding more resources.
    - Calculate the time savings (TS) = TN - TC for each critical activity.
3. **Determine Cost of Crashing:**
    - Calculate the cost per unit time saved for each critical activity. This cost includes additional resources, overtime, or penalties for late completion.
    - Cost per Unit Time Saved (CUTS) = Cost of crashing / Time savings (TS).
4. **Identify Optimal Crashing Strategy:**
    - Determine the activity with the lowest CUTS value. This activity should be crashed first to achieve maximum time savings at minimum cost.
    - Continue identifying and crashing activities in order of increasing CUTS value until the desired project duration is achieved or the cost becomes prohibitive.

5. **Update Project Schedule and Costs:**
   - Update the project schedule to reflect the crashed activities and their new durations.
   - Calculate the total cost of the project after crashing. This includes both regular project costs and the additional cost incurred by crashing.
6. **Monitor Progress:**
   - Continuously monitor the progress of the project to ensure that crashed activities are completed as per the new schedule.
   - Keep track of actual costs incurred due to crashing compared to the planned costs.

Example of Crashing a Project in PERT:

Consider a construction project with the following critical activities on the critical path:

Acti	Normal Time (	Crash Time (	Time Savings (	Cost of Crashing	CUTS ($/day sav
A	5 days	3 days	2 days	$5000	$2500
B	4 days	2 days	2 days	$3000	$1500
C	3 days	2 days	1 day	$2000	$2000
D	6 days	4 days	2 days	$4000	$2000

Assuming the project needs to be completed in 12 days and the current schedule totals 18 days:

- Total time to crash = 18 days - 12 days = 6 days.
- Total cost to crash = $5000 + $3000 + $2000 + $4000 = $14000.

The project manager decides to crash Activity A and Activity C first:

- Time savings from A = 2 days * 2 activities = 4 days.
- Time savings from C = 1 day * 1 activity = 1 day.

Total time savings = 4 days + 1 day = 5 days.

Cost per day saved (CUTS) = $14000 / 5 days = $2800/day.

The project manager then proceeds to crash Activity D:

- Time savings from D = 2 days * 1 activity = 2 days.

Total time savings = 5 days + 2 days = 7 days.

Cost per day saved (CUTS) = $14000 / 7 days = $2000/day.

By following this crashing strategy, the project can be completed in 12 days with a total crashing cost of $14000, ensuring that the deadline is met while minimizing additional expenses.

5.4 SCHEDULING A PROJECT

Scheduling a project using PERT (Program Evaluation and Review Technique) involves creating a detailed plan that identifies the sequence of activities, their estimated durations, dependencies, and critical path. Here are the steps to schedule a project using PERT:

Steps to Schedule a Project Using PERT:

1. **Identify Project Tasks/Activities:**
   - Break down the project into smaller tasks or activities that need to be completed.
   - Ensure that each activity is well-defined and has a specific start and end point.
2. **Define Activity Dependencies:**
   - Determine the dependencies between activities. Some activities may need to be completed before others can start.
   - Represent dependencies using a network diagram, with arrows showing the flow of activities.
3. **Estimate Activity Durations:**
   - Assign time estimates to each activity based on historical data, expert judgment, or similar past projects.
   - Use three time estimates for each activity: Optimistic (O), Most Likely (M), and Pessimistic (P).

4. **Calculate Expected Activity Durations:**
   - Calculate the expected duration (TE) for each activity using the formula: TE=O+4M+P6TE = \frac{{O + 4M + P}}{6}TE=6O+4M+P
   - This formula gives more weight to the most likely estimate (M) while considering the optimistic (O) and pessimistic (P) estimates.
5. **Create a Network Diagram:**
   - Develop a visual representation of the project activities and their dependencies using a network diagram.
   - Use nodes to represent activities and arrows to show the sequence and dependencies between activities.
6. **Identify Critical Path:**
   - Determine the critical path, which is the longest path through the network diagram in terms of expected duration.
   - Activities on the critical path have zero slack or float, meaning any delay in these activities will delay the entire project.
7. **Calculate Total Project Duration:**
   - Sum up the expected durations of activities on the critical path to calculate the total project duration.
   - The total project duration represents the minimum time required to complete the project.
8. **Identify Slack/Float Time:**
   - Calculate slack or float time for non-critical activities. Slack time is the amount of time an activity can be delayed without delaying the project's overall completion.
   - Slack time = Late Start (LS) - Early Start (ES) or Late Finish (LF) - Early Finish (EF).

9. **Optimize Resource Allocation:**
   - Review resource availability and allocation for each activity to ensure resources are efficiently utilized.
   - Adjust resource assignments or schedules as needed to optimize project performance.
10. **Update Schedule and Monitor Progress:**
    - Update the project schedule with the calculated durations, dependencies, and critical path.
    - Continuously monitor project progress, track actual versus planned durations, and make adjustments as necessary to keep the project on schedule.

Example:

Consider a software development project with the following activities and estimated durations:

Acti	Optimistic	Most Likely	Pessimistic	Dependen
A	4 days	6 days	8 days	-
B	3 days	5 days	7 days	A
C	2 days	4 days	6 days	A
D	5 days	7 days	9 days	B, C
E	6 days	8 days	10 days	C
F	3 days	4 days	5 days	D, E

Steps:

1. **Calculate Expected Durations (TE) for Each Activity:**
   - Activity A: TEA=4+4(6)+86=6TE_A = \frac{4 + 4(6) + 8}{6} = 6TEA=64+4(6)+8=6 days
   - Activity B: TEB=3+4(5)+76=5TE_B = \frac{3 + 4(5) + 7}{6} = 5TEB=63+4(5)+7=5 days

- Activity C: TEC=2+4(4)+66=4TE_C = \frac{2 + 4(4) + 6}{6} = 4TEC=62+4(4)+6 =4 days
- Activity D: TED=5+4(7)+96=7TE_D = \frac{5 + 4(7) + 9}{6} = 7TED=65+4(7)+9 =7 days
- Activity E: TEE=6+4(8)+106=8TE_E = \frac{6 + 4(8) + 10}{6} = 8TEE =66+4(8)+10=8 days
- Activity F: TEF=3+4(4)+56=4TE_F = \frac{3 + 4(4) + 5}{6} = 4TEF=63+4(4)+5=4 days

2. **Create a Network Diagram:**

plaintext

Copy code

```
A --> B --> D --> F
 \      /
  --> C --> E
```

3. **Identify Critical Path:**

Critical Path: A-B-D-F with an expected duration of 6+5+7+4=226 + 5 + 7 + 4 = 226+5+7+4=22 days.

4. **Calculate Total Project Duration:**

Total project duration = 22 days.

5. **Identify Slack Time:**

- Activity A: Slack = 0 days
- Activity B: Slack = 0 days
- Activity C: Slack = 1 day (LS = 6, ES = 5)
- Activity D: Slack = 1 day (LS = 22, ES = 21)
- Activity E: Slack = 0 days
- Activity F: Slack = 0 days

6. **Optimize Resource Allocation:**

Ensure resources are allocated efficiently to activities on the critical path to avoid delays in project completion.

7. **Update Schedule and Monitor Progress:**

Update the project schedule with the calculated durations and critical path. Monitor progress regularly and make adjustments as needed to meet project deadlines.

By following these steps, you can effectively schedule and manage a project using PERT, ensuring that activities are completed on time and within budget.

Unit summary

Network analysis in operations research is centered on optimizing and managing complex networks to improve operational efficiency across various domains such as logistics, transportation, supply chain management, and project scheduling. This unit encompasses key methodologies and concepts, including network flow models, shortest path algorithms, maximum flow problems, and project management techniques like the Critical Path Method (CPM) and Program Evaluation and Review Technique (PERT). By studying these topics, students gain the analytical tools to design, analyze, and optimize networks, enabling effective decision-making and enhanced operational performance. Practical applications and real-world examples illustrate how network analysis can solve complex problems, providing a comprehensive understanding of how to implement these strategies in diverse industrial and organizational settings.

Glossary

1. **Critical Path Method (CPM)**: A project management technique used to determine the sequence of activities that have the longest duration in a project. The critical path dictates the shortest possible project duration, and any delay in these activities will delay the entire project.
2. **Program Evaluation and Review Technique (PERT)**: A statistical tool used in project management to analyze and represent the tasks involved in completing a given project. PERT focuses on the time needed to complete each task and the minimum time required to complete the total project.

3. **Node**: A fundamental element of network diagrams representing events, milestones, or tasks. In project management, nodes are used to mark the start and end points of tasks or activities in a project network.
4. **Arc (or Edge)**: In network analysis, an arc represents a directed connection between two nodes. It often signifies the flow of resources, tasks, or dependencies in a project network, indicating the sequence and relationship between activities.
5. **Slack Time**: The amount of time that an activity can be delayed without delaying the overall project completion time. Calculating slack time helps project managers identify which activities have flexibility in their scheduling and which are critical to maintaining the project timeline.

Self Assessment Problems

1. Activity A has an optimistic time of 4 days, a most likely time of 6 days, and a pessimistic time of 8 days. Calculate the expected time for Activity A using PERT.

2. Activity B has an optimistic time of 3 days, a most likely time of 5 days, and a pessimistic time of 7 days. Calculate the variance for Activity B using PERT.

3. A project has three activities with the following time estimates (O, M, P): Activity X (5, 7, 9), Activity Y (6, 8, 10), Activity Z (4, 5, 6). Calculate the expected duration and variance for each activity using PERT.

4. Calculate the expected duration of a project with activities A (4, 6, 8), B (3, 5, 7), and C (2, 4, 6) using PERT.

5. A project consists of four activities: A (3, 5, 7), B (2, 4, 6), C (5, 7, 9), and D (4, 6, 8). Determine the critical path and the total expected duration using PERT.

6. Activity A takes 6 days, Activity B takes 4 days, and Activity C takes 5 days to complete. Determine the critical path and the total project duration using CPM.

7. A project has five activities with the following durations: A (3 days), B (5 days), C (2 days), D (6 days), E (4 days). Determine the critical path and the total project duration using CPM.

8. Activity X has a normal time of 8 days and a crash time of 5 days. Calculate the crash cost per day for Activity X using CPM.

9. A project has four activities with the following durations and costs: A (2 days, $100/day), B (3 days, $150/day), C (4 days, $200/day), D (5 days, $250/day). Determine the optimal crashing strategy and the total project cost using CPM.
10. Activity Y has a normal time of 6 days and a crash time of 4 days. The crash cost per day is $200. Determine the total cost to crash Activity Y using CPM.
11. A project has activities A (3, 4, 5) and B (2, 3, 4). Calculate the expected duration and variance for each activity using PERT, then determine the critical path and total project duration using CPM.
12. A project consists of activities X (5, 6, 7), Y (4, 5, 6), and Z (3, 4, 5). Calculate the expected duration and variance for each activity using PERT, then determine the critical path and total project duration using CPM.
13. Activity M has a normal time of 10 days and a crash time of 6 days. The crash cost per day is $300. Activity N has a normal time of 8 days and a crash time of 5 days. The crash cost per day is $250. Determine the optimal crashing strategy and the total project cost using PERT and CPM.
14. A project has activities P (4, 6, 8) and Q (3, 5, 7). Calculate the expected duration and variance for each activity using PERT, then determine the critical path and total project duration using CPM.
15. A project consists of activities R (2, 3, 4) and S (5, 7, 9). Calculate the expected duration and variance for each activity using PERT, then determine the critical path and total project duration using CPM.
16. Activity A has an optimistic time of 3 days, a most likely time of 5 days, and a pessimistic time of 7 days. Activity B depends on A and has an optimistic time of 2 days, most likely time of 4 days, and pessimistic time of 6 days. Determine the expected duration and critical path for the project.

Exercise

17. A construction project consists of the following activities:

- Activity A: 4 days
- Activity B (depends on A): 5 days
- Activity C (depends on A): 3 days
- Activity D (depends on B and C): 6 days Calculate the critical path and total project duration.

18. A has an optimistic time of 2 weeks, a most likely time of 3 weeks, and a pessimistic time of 4 weeks. Activity B depends on A and has an optimistic time of 1 week, most likely time of 2 weeks, and pessimistic time of 3 weeks. Determine the expected duration and variance for each activity.

19. A software development project has the following activities:

- Activity A: 3 days
- Activity B (depends on A): 4 days
- Activity C (depends on A): 2 days
- Activity D (depends on B and C): 5 days Determine the critical path and total project duration.

20. Activity A has an optimistic time of 4 days, a most likely time of 6 days, and a pessimistic time of 8 days. Activity B depends on A and has an optimistic time of 3 days, most likely time of 5 days, and pessimistic time of 7 days. Determine the expected duration and variance for each activity.

21. A marketing campaign project includes the following activities:

- Activity A: 2 weeks
- Activity B (depends on A): 3 weeks
- Activity C (depends on A): 2 weeks
- Activity D (depends on B and C): 4 weeks Calculate the critical path and total project duration.

22. Activity A has an optimistic time of 6 days, a most likely time of 8 days, and a pessimistic time of 10 days. Activity B depends on A and has an optimistic time of 5 days, most likely time of 7 days, and pessimistic time of 9 days. Determine the expected duration and variance for each activity.

23. A construction project consists of the following activities:

- Activity A: 5 days
- Activity B (depends on A): 6 days
- Activity C (depends on A): 4 days
- Activity D (depends on B and C): 7 days Calculate the critical path and total project duration.

24. Activity A has an optimistic time of 1 month, a most likely time of 2 months, and a pessimistic time of 3 months. Activity B depends on A and has an optimistic time of 2 weeks, most likely time of 4 weeks, and pessimistic time of 6 weeks. Determine the expected duration and variance for each activity.

25. A manufacturing project includes the following activities:

- Activity A: 4 days
- Activity B (depends on A): 3 days
- Activity C (depends on A): 5 days
- Activity D (depends on B and C): 4 days Calculate the critical path and total project duration.

26. Activity A has an optimistic time of 2 weeks, a most likely time of 4 weeks, and a pessimistic time of 6 weeks. Activity B depends on A and has an optimistic time of 1 week, most likely time of 2 weeks, and pessimistic time of 3 weeks. Determine the expected duration and variance for each activity.

27. A construction project consists of the following activities:

- Activity A: 3 weeks
- Activity B (depends on A): 4 weeks

- Activity C (depends on A): 2 weeks
- Activity D (depends on B and C): 5 weeks Calculate the critical path and total project duration.

28. Activity A has an optimistic time of 5 days, a most likely time of 7 days, and a pessimistic time of 9 days. Activity B depends on A and has an optimistic time of 4 days, most likely time of 6 days, and pessimistic time of 8 days. Determine the expected duration and variance for each activity.

29. A software development project includes the following activities:

- Activity A: 2 months
- Activity B (depends on A): 3 months
- Activity C (depends on A): 2 months
- Activity D (depends on B and C): 4 months Calculate the critical path and total project duration.

30. Activity A has an optimistic time of 3 days, a most likely time of 5 days, and a pessimistic time of 7 days. Activity B depends on A and has an optimistic time of 2 days, most likely time of 4 days, and pessimistic time of 6 days. Determine the expected duration and variance for each activity.

31. A marketing campaign project includes the following activities:

- Activity A: 4 weeks
- Activity B (depends on A): 5 weeks
- Activity C (depends on A): 3 weeks
- Activity D (depends on B and C): 6 weeks Calculate the critical path and total project duration.

32. Activity A has an optimistic time of 6 days, a most likely time of 8 days, and a pessimistic time of 10 days. Activity B depends on A and has an optimistic time of 5 days, most likely time of 7 days, and pessimistic time of 9 days. Determine the expected duration and variance for each activity.

33. A construction project consists of the following activities:

- Activity A: 5 weeks
- Activity B (depends on A): 6 weeks
- Activity C (depends on A): 4 weeks
- Activity D (depends on B and C): 7 weeks Calculate the critical path and total project duration.

Check Your Progress Answer

A) Dijkstra's algorithm
B) Ford-Fulkerson algorithm
C) Scheduling and managing complex projects
B) The longest path through the network with the shortest completion time
C) Estimate the project duration considering uncertainty

Suggested Readings

1. Introduction to Operations Research" by Frederick S. Hillier and Gerald J. Lieberman
2. Operations Research: An Introduction" by Taha H.A.
3. Operations Research: Applications and Algorithms" by Wayne L. Winston
4. Network Flows: Theory, Algorithms, and Applications" by Ravindra K. Ahuja, Thomas L. Magnanti, and James B. Orlin

Open Source-E Content Link

1. https://www.youtube.com/watch?v=_zhtDJX3kOY
2. https://www.youtube.com/watch?v=27MpB4nd3EI
3. https://www.youtube.com/watch?v=eKSCejfnsmk
4. https://www.youtube.com/watch?v=M8POtpPtQZc
5. https://www.youtube.com/watch?v=FYqg62rYxhs

References

1. C.R.Kothari, ―Quantitative Techniques‖, VikasPublications, Noida
2. V.K.Kappor,"OperationsResearch-ProblemsandSolutions",Sultan Chand&SonsPublisher,NewDelhi
3. AnandSharma,OperationResearch,HimalayaPublishingHouse,2014, Mumbai